2.3 Correlation functions

We would like to extract the critical exponents η, ν from the propagator,

$$G(x,y) = 1/\frac{\delta^2\Gamma}{\delta M^2}(x,y) \qquad (2.39)$$

$$\left[\overset{\shortparallel}{\partial B(x)/\partial M(y)}\right]$$

The second functional derivative of Γ reads

$$\frac{\delta^2\Gamma}{\partial M^2}(x,y) = \left[-\partial_\mu^{x\,2} + r_0(T) + \frac{1}{2}u_0 M^2(x)\right]\delta(x-y) \qquad (2.40)$$

or, in momentum space,

$$\widetilde{\frac{\delta^2\Gamma}{\partial M^2}}(p,q) = \left(p^2 + r_0(T) + \frac{1}{2}u_0 M^2\right)\delta(p-q)$$

This leads to the propagator, $\tilde{G}(p,q) = \tilde{G}(p)\,\delta(p-q)$

$$\boxed{\tilde{G}(p) = \frac{1}{p^2 + r_0(T) + 1/2\, u_0 M^2}} \qquad (2.41)$$

where, for $M = M_0$, one sees explicitly the relation between the critical exponents

Now we evaluate the propagator for $M = M_0$.

i) For $\boxed{T > T_c}$ we have $M_0 = 0$ and hence

$$\tilde{G}(p) = \frac{1}{p^2 + \underbrace{r_0(T_c - T)}_{1/\xi^2}} \simeq \frac{1}{(p^2)^{1-\eta/2}} f(p \cdot \xi) \qquad (2.42)$$

$$\xi = [r_0(T_c - T)]^{-1/2}$$

see eq. (1.76), p. 36. It follows

	Landau	d=2	d=3	
η	0	1/4	0.0357 ± 0.0025	(2.43)
			0.6305 ± 0.0015	
ν	1/2	1		

ii) For $\boxed{T < T_c}$ we have $M_0^2 = \frac{6 r_0}{u_0}(T_c - T)$, see eq. 2.35, p. 55

and hence

$$\tilde{G}(p) = \frac{1}{p^2 + \underbrace{(2)(T_c - T)}_{1/\xi^2}} \qquad (2.44)$$

$$\xi = [(2) r_0 (T - T_c)]^{-1/2}$$

again leading to (2.43). Note also the relative slopes for $\tilde{G}_{T \gtrless T_c}(p=0) \sim \chi$, see p. 23-24.

The propagator in position space follows easily as

$$G(r) = \frac{1}{4\pi r} e^{-r/\xi} \qquad (2.45)$$

Exercise