4.4. RG-equations, anomalous dimensions & critical phenomena

In the last chapters we have introduced the functional RG for the effective action, studied pert. theory, and derived critical exponents within the effect. potential approx. Here we want to formalise our findings.

Let us start again with the flow equation, eq. (4.51), p. 123:

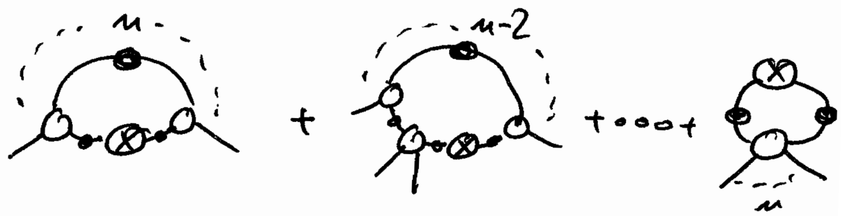
$$\boxed{\partial_t \Gamma_k[\phi] = \frac{1}{2} \operatorname{Tr} \frac{1}{\Gamma_k^{(2)}[\phi]+R_k} \partial_t R_k} \qquad (4.114)$$

We have introduced Γ as the Legendre transform of the logarithm of $\ln Z$, and so far we have only used the fact, that the propagator G is related to $\Gamma^{(2)}$: $G = 1/\Gamma^{(2)}$. This entails that $\Gamma^{(2)}$ is a one-particle irreducible (1PI) Green function. Indeed, Γ is the gen. functional of 1PI Green fcts. This follows readily from the flow: Assume $\Gamma_k[\phi]$ is 1PI. Then

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma^{(2)} + R_k} \partial_t R_k$$

and

$$\dot\Gamma_k^{(n)}[\phi] =$$



All these diagrams are 1PI, and so is

$$\Gamma_{k_1} = \Gamma_k + \int_k^{k_1} \frac{dk'}{k'} \dot\Gamma_{k'} \qquad (4.115)$$

We conclude that if the initial effective action is 1PI (e.g. the classical action), the flow transports this property to all k.

In pert. theory we have learned that a renormalisable theory only has a finite number of relevant ops., in the ϕ^4-theory this is $(\partial_\nu\phi)^2$, ϕ^2, and ϕ^4 with the 'coupling' constants Z, m^2, and λ. These constants are usually fixed with ($m^2 \neq 0$)

$$\Gamma^{(2)}(p^2=0) = m^2_{phys.}$$
$$\partial_{p^2}\Gamma^{(2)}(p^2)\Big|_{p^2=0} = 1 \qquad (4.116)$$
$$\Gamma^{(4)}(p_i=0) = \lambda_{phys.} \qquad \mu^2=0$$

μ: renormalisation scale

In the flow setting eq. (4.116) translates into similar conditions for $\Gamma^{(n)}_\Lambda$ at the initial scale Λ. We also rewrite the flow in the spirit of our dim. less flow in the last chapter.

The standard procedure is now to use the invariance of the bare action/Green function of the renormalisation scale μ in order to derive RG-equations for the renormalised quantities. Here this translates to the invariance of $\Gamma_{k=0}$ under a variation of the initial scale Λ.

Let us now take a view point in between: The full, renormalised effective action $\Gamma_{k=0}[\phi]$ is invariant under a change of μ. This only amounts to changing the renormalisation point,

$$\boxed{\mu \frac{d}{d\mu} \Gamma[\phi] = 0} \qquad (4.117)$$

However, this implies that fields and couplings are scaled accordingly, to wit

$$\mu \frac{d}{d\mu} \phi = -\frac{1}{2} \gamma_\phi \phi \,, \quad \mu \frac{d}{d\mu} \ln m^2 = -\gamma_m \,, \quad \mu \frac{d}{d\mu} \lambda = \beta \qquad (4.118)$$

Anomalous dimensions

leading to the RG-equation

$$\boxed{\left(\mu\frac{\partial}{\partial\mu} + \beta\,\partial_\lambda - \gamma_m m^2 \partial_{m^2} - \frac{1}{2}\gamma_\phi \int_x \phi \frac{\delta}{\delta\phi}\right)\Gamma = 0} \quad (4.119)$$

or for Green fcts. $\Gamma^{(n)} = \frac{\delta^n}{\delta\phi^n}\Gamma\Big|_{\phi=0}$:

$$\boxed{\left(\mu\frac{\partial}{\partial\mu} + \beta\,\partial_g - \gamma_m m^2 \partial_{m^2} - \frac{n}{2}\gamma_\phi\right)\Gamma^{(n)} = 0} \quad (4.120)$$

Remarks:

(i) The relative minus signs in eq. (4.118)-(4.120) stem from their original derivation from bare quantities. They are just conventions.

(ii) The partial μ-derivative is at fixed λ, m, ϕ. One can extend and unify this notation by defining

$$\beta_i = \mu\frac{d}{d\mu}\lambda_i \quad \text{with } \vec{\lambda} = (m, \lambda, Z, \ldots)$$

and hence

$$\boxed{\left(\mu\frac{\partial}{\partial\mu} + \vec{\beta}\cdot\vec{\partial}_\lambda - \frac{n}{2}\gamma_\phi\right)\Gamma^{(n)} = 0} \quad (4.121)$$

(iii) The flow equation eq. (4.114) can be combined with eq. (4.119), the latter describing a reparametrisation of the theory, the former an integrating-out of dofs, hence a physical change. For the sake of simplicity we set $\mu = k$ and are let to

$$\boxed{\left(k\partial_k + \beta \partial_\lambda - \gamma_m m^2 \partial_{m^2} - \gamma_\phi \int_x \phi \frac{\delta}{\delta \phi} \right) \Gamma_k = \frac{1}{2} \operatorname{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \dot{R}_k} \tag{4.122}$$

Remarks:

(i) Note that the $\gamma_m \, m^2 \partial_{m^2}$-term and the flow on the rhs are similar, indeed, for $R_k = k^2$ it produces the canonical running of the mass term.

(ii) Note also that the lhs in eq. (4.122) accounts for the $\boxed{\text{anomalous}}$ scaling!

Running coupling:

let us look at the scale-dependence of the 4-point function (in a mass-less theory)

$$(\mu \partial_\mu + \beta \partial_\lambda - 2\gamma_\phi)\, \Gamma^{(4)} = 0 \qquad (4.123)$$

It is clear from eq. (4.123) that $\Gamma^{(4)}$ is <u>not</u> RG-invariant, $\mu \frac{d}{d\mu} \Gamma^{(4)} = 2\gamma_\phi \Gamma^{(4)}$, originated in the μ-dep. of ϕ. An invariant combination is

$$\hat{\Gamma}^{(4)}(p_i ; \lambda, \mu) = \Gamma^{(4)}(p_i; \lambda, \mu) \Big/ \prod_{i=1}^{4} \frac{\Gamma^{(2)}(p_i; \lambda, \mu)}{p_i^2} \qquad (4.124)$$

where $\Gamma^{(2)}(p_i, \lambda, \mu)/p_i^2$ is nothing but our Z_k $(+\, \partial_\phi^2 V(\phi=0))$, the momentum-dep wave function renormalisation or (inverse) dressing fct.

$\hat{\Gamma}^{(4)}$ satisfies $\mu \frac{d}{d\mu} \hat{\Gamma}^{(4)} = 0$, or,

$$\boxed{(\mu \partial_\mu + \beta \partial_\lambda)\, \hat{\Gamma}^{(4)} = 0} \qquad (4.125)$$

$\hat{\Gamma}^{(4)}$, evaluated at some conveniently chosen momentum configuration, e.g. the symmetric point, is called the running coupling, not to be confused with the renormalised coupling. Its scale- or momentum-dependence is stored in the β-fct. $\beta(g)$.

We have already computed it at one loop:

There, Z is trivial, $Z(\lambda)=1$, as the self-energy $\Gamma^{(2)}_{1\text{-loop}} - S^{(2)}$ is momentum-independent. The 1-loop running of $\Gamma^{(4)}_k$ was computed on p. 127d, eq. (4.73) and reads ($\lambda/4!\,\phi^4$)

$$\partial_t \Gamma^{(4)}(p_i=0) = \frac{3}{16\pi^2}\lambda^2 \qquad (4.126)$$

$\boxed{d=4}$

to be compared with

$$\beta\Big|_{1\text{-loop}} = -\left.\frac{\mu\partial_\mu \hat\Gamma^{(4)}(p_i=0)}{\underbrace{\partial_\lambda \hat\Gamma^{(4)}(p_i=0)}_{=\,1+\mathcal{O}(\lambda^2)}}\right|_{1\text{-loop}} = -\mu\partial_\mu \hat\Gamma^{(4)}(p_i=0) \qquad (4.127)$$

As $\hat\Gamma^{(4)}(p_i=0)$ is dim.less, it can only depend on the ratio μ/k, and hence $\mu\partial_\mu\hat\Gamma^{(4)} = -\partial_t \hat\Gamma^{(4)}$.

We conclude
$$\beta\Big|_{1\text{-loop}} = \partial_t \hat{\Gamma}^{(4)}(p_i=0)\Big|_{1\text{-loop}} = \frac{3}{16\pi^2}\lambda^2 \qquad (4.128)$$

We emphasise that the equivalence between μ- and k-scaling is, in general, not present beyond one-loop. To see this, let us go to another RG-scheme, where the renormalised coupling λ is changed (λ dimless)

$$\begin{aligned} \lambda' &= \lambda + c_1 \lambda^2 + \dots \\ \Rightarrow \lambda &= \lambda' - c_1 \lambda'^2 + \dots \end{aligned} \qquad (4.129)$$

The β-fct. follows from the second line in (4.129) as

$$\beta = \beta' - 2c_1 \beta' \lambda' + \dots \qquad (4.130)$$

with

$$\beta = \underbrace{\beta_0}_{= \frac{3}{16\pi^2}} \lambda^2 + \beta_1 \lambda^3 + \dots \qquad (4.131)$$

Inserting eq. (4.131) in (4.130) leads us to

$$\beta_0 \lambda^2 + \beta_1 \lambda^3 + \dots = \beta_0' \lambda'^2 + \beta_1' \lambda'^3 - 2c_1 \lambda' (\beta_0' \lambda'^2 + \dots)$$

$$= \beta_0' \lambda^2 + \cancel{2 \beta_0' c_1 \lambda^3} + \beta_1' \lambda^3 - \cancel{2 c_1 \beta_0' \lambda^3} + O(\lambda^5)$$

Comparing the λ-orders gives us (4.132)

$$\beta_0 = \beta_0' , \quad \beta_1 = \beta_1' = \frac{17}{3} \frac{1}{(16\pi^2)^2} \qquad (4.133)$$

but

$$\beta_i \neq \beta_i' \quad i > 1 \; ! \qquad (4.134)$$

The β-fct. is 2-loop <u>universal</u>. Indeed one can show that in general the first two non-vanishing coefficients are <u>universal</u>.

Exercise

Remark: The above holds true only if no other scales are present, i.e. no mass scale. 1-loop universality holds even then, but 2-loop universality breaks down.

The above result is potentially alarming. As the coupling(s) are not universal, what about a fixed point. To that end let us study the fixed point eq. again, $\boxed{\vec{\lambda} \text{ dimless}}$

$$\vec{\beta}(\vec{\lambda}_*) = \mu \partial_\mu \vec{\lambda} \Big|_{g=g_*} = 0 \qquad (1.135)$$

We reparameterise our theory, $\vec{\lambda} = \vec{\lambda}(\vec{\lambda}')$, and hence, $\vec{\lambda}_* = \vec{\lambda}(\vec{\lambda}'_*)$, where $\vec{\lambda}'_*$ is that vector which gives $\vec{\lambda}_*$. Inserting this into eq. (1.135) gives

$$\mu \partial_\mu \lambda_i(\vec{\lambda}')\Big|_{\vec{\lambda}'_*} = \left(\frac{\partial \lambda}{\partial \lambda'}\right)_{ij} \cdot \mu \partial_\mu \lambda'_j \qquad (1.136)$$

If the map $\vec{\lambda}' \to \vec{\lambda}$ is well-defined, the Jacobian $\left(\frac{\partial \lambda}{\partial \lambda'}\right)$ has no zero modes, $\det \frac{\partial \lambda}{\partial \lambda'} \neq 0$, and we conclude

$$\boxed{\mu \partial_\mu \lambda'_j \Big|_{\vec{\lambda}'_*} = 0} \qquad (1.137)$$

Full running coupling: Now we are in the position to give a closed expression of $\Gamma^{(4)}(p_i;\lambda,\mu)$ as a function of momentum. Firstly, multiplying μ with a scale factor s leads us to, $\mu \to s\mu = \mu(s)$, $\boxed{\lambda \text{ dimless}}$

$$s\frac{d}{ds}\hat{\Gamma}^{(4)}(p_i;\lambda(s),\mu(s)) = 0 \qquad (1.138)$$

and hence

$$\boxed{\Gamma^{(4)}(p_i;\lambda(s),\mu(s)) = e^{2\int_{\lambda}^{\lambda(s)}\frac{\gamma(\lambda')}{\beta(\lambda')}d\lambda'}\Gamma^{(4)}(p_i;\lambda,\mu)} \qquad (1.139)$$

Eq. (1.139) generalises to

$$\boxed{\Gamma^{(n)}(p_i;\lambda(s),\mu(s)) = e^{\left[n/2\int_{\lambda}^{\lambda(s)}\frac{\gamma(s')}{\beta(s')}d\lambda'\right]}\Gamma^{(n)}(p_i;\lambda,\mu)} \qquad (1.140)$$

For $s=1$ we have $\mu(s)=\mu\cdot s=\mu$, and $\lambda(1)=\lambda$. Then the integrals in eq. (1.139), (1.140) are trivially 1. The $s\frac{d}{ds}$-derivative gives: $\boxed{s\frac{d}{ds}\Gamma^{(n)} = \frac{n}{2}\gamma\,\Gamma^{(n)} \quad (1.141)}$

We are primarily interested in the physical momentum behaviour. We shall see that it is encoded in the anomalous dimensions $\vec{\beta}$ $(\beta, \gamma, \ldots)$. To see this we use that the <u>canonical</u> dimensio of $\Gamma^{(n)}$ is $d - n[\phi]$ with $[\phi] = \frac{d-2}{2}$, see p. 155a

This entails

dim. less

$$\boxed{\Gamma^{(n)}(sp_i;\lambda, s\mu) = s^{d_n}\,\Gamma^{(n)}(p_i;\lambda,\mu)} \quad (4.142)$$

with $d_n = d - n\left(\frac{d-2}{2}\right)$. Together with eq.(1.13 this leads to

$$\boxed{\Gamma^{(n)}(s\,p_i;\lambda,\mu) = s^{d-n\left(\frac{d-2}{2}\right)}\, e^{-\frac{n}{2}\int_{\lambda}^{\lambda(s)} \frac{\gamma(\lambda')}{\beta(\lambda')}d\lambda'} \cdot \Gamma^{(n)}(p_i;\lambda(s),\mu)} \quad (4.143)$$

With eq.(4.143) we have mapped the momentum scaling to the anomalous dimensions.

Dim of $\Gamma^{(n)}$:

$$\Gamma^{(n)}(p_i; \lambda, \mu) \cdot (2\pi)^d \delta(p_1 + \dots + p_n)$$

$$= \frac{\delta^n \Gamma}{\delta\phi(p_1) \cdots \delta\phi(p_n)} \qquad (4.144)$$

(1) From $\frac{\delta}{\delta\phi(p)} \phi(q) = \delta(p-q)$ it follows

that $\left[\frac{\delta}{\delta\phi(p)}\right] = -d - [\phi(q)] = -d - \left(\frac{d-2}{2} - d\right)$

$$\qquad\qquad\qquad\qquad\qquad \overset{\shortparallel}{[\phi(x)]} \qquad (4.145)$$

$$= [\phi(x)]$$

(2) From $[\Gamma^{(n)}(p)] = \left[\frac{\delta^n}{\delta\phi(p)^n} \Gamma\right] + d$, see eq. (4.144)

and eq. (4.145) it follows that

$$[\Gamma^{(n)}(p)] = d - n[\phi(x)] = d - n\left(\frac{d-2}{2}\right) \qquad (4.146)$$

Note that μ has dropped out of the scaling relation eq. (4.143); it is a spectator.

Now we recall our result for the one-loop β-fct. in $d=4$:

$$\beta = \frac{\overset{\beta_0}{\overset{\shortparallel}{3}}}{16\pi^2}\,\overset{\text{dim. less}}{\overset{\downarrow}{\lambda}}{}^2 \qquad (4.147)$$

with the Gaußian FP $\lambda = 0$. For

$$\boxed{\varepsilon = 4 - d} \qquad (4.148)$$

the dimensionfull coupling has the dimension ε. Hence the β-fct. compensates for this factor, see explicitly in eq. (4.99), p. 138,

$$\beta = -\varepsilon\lambda + \frac{3}{16\pi^2}\lambda^2 + \mathcal{O}(\lambda^3) \qquad (4.149)$$

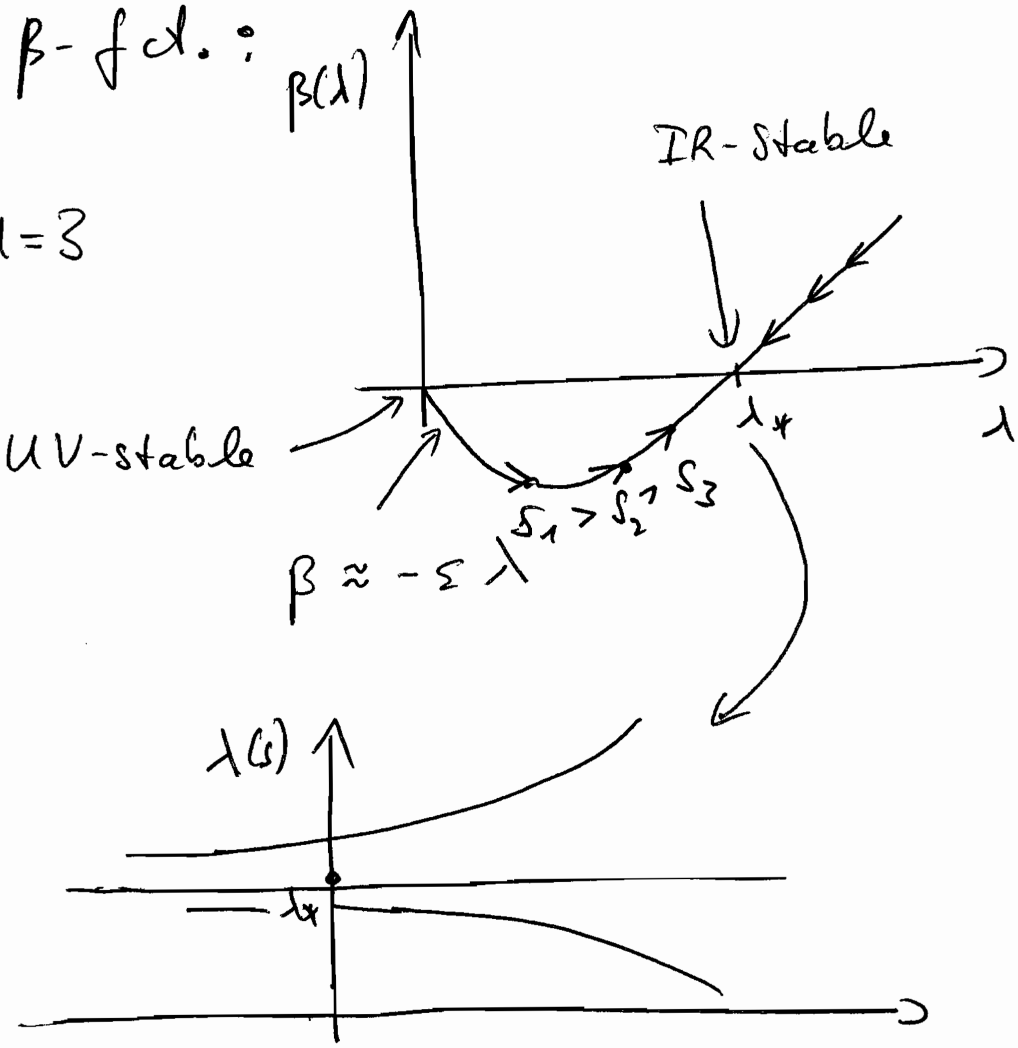
leading to

$$\boxed{\lambda_* = \frac{16\pi^2\varepsilon}{3}} \qquad (4.150)$$

see also eq. (4.99) ff.

β-fct.:

$d=3$



and, similarily for UV-stability, see also p. 86.

Assume now that we are in the vicinity of an IR-attractive fixed point. This FP is reached for $s \to 0$:

$$s \frac{d\lambda}{ds} = \beta \implies \ln s/s_0 = \int_{\lambda}^{\lambda(s)} \frac{d\lambda'}{\beta(\lambda')} \qquad (4.151)$$

For $\lambda > \lambda_*$ we have $\beta > 0$, and hence $\ln s/s_0 < 0$. if approaching the FP. For $\lambda < \lambda_*$ we have $\beta < 0$, and hence $\ln s/s_0 < 0$ if approaching the FP.

Accordingly,

$$\lim_{s\to 0} \int_{\lambda}^{\lambda(s)} \frac{\gamma(\lambda')}{\beta(\lambda')} d\lambda' = \lim_{s\to 0} \int_{0}^{\ln s} \gamma(\lambda(s')) \frac{ds'}{s'} \approx \gamma(\lambda_*) \ln s \qquad (4.152)$$

and hence for the (inverse) propagator

$$\boxed{\lim_{s\to 0} \Gamma^{(2)}(s\,p; \lambda, \mu) \sim s^{2-\gamma(\lambda_*)}\, \Gamma^{(2)}(p; \lambda_*, \mu)} \qquad (4.153)$$

This entails that

$$\boxed{\eta = \gamma(\lambda_*)} \tag{4.154}$$

which we have already used implicitly in the local potential approximation of the flow equation.

Eq. (4.151) relates one of the two indep. critical exponents (η, ν) to the anomalous dimension of the field, γ_ϕ. The exponent ν relates to the mass parameter in the propagator. In the presence of a massive field (or $T > T_c$) we have

$$\Big(\mu\,\partial_\mu + \beta\,\partial_\lambda - \gamma_{m^2}\, \overset{\text{dim. less}}{\overset{\downarrow}{m^2}}\, \partial_{m^2} - \frac{n}{2}\gamma_\phi\Big)\Gamma^{(n)} = 0 \tag{4.155}$$

eq. (4.120).

Note that the renormalisation encoded in $\beta, \gamma_{m^2}, \gamma_\phi$ can be set-up in a mass-indep. way. To that end we write

$$\Gamma^{(n)}(p_i; \lambda, m^2, \mu) = \sum_\ell \frac{m^2}{\ell!} \Gamma^{(n,\ell)}(p_i, \lambda, 0, \mu)$$

(4.158)

with

$$\Gamma^{(n,\ell)}(p_i; \lambda, 0, \mu) = \partial_{m^2} \Gamma^{(n,\ell-1)}(p_i, \lambda, m^2, \mu)\Big|_{m^2=0}$$

Assuming convergence, eq. (4.153) defines $\Gamma^{(n)}$.

The $\Gamma^{(n,\ell)}$ satisfy the RG-equations (CS-eq.)

$$\boxed{\left(\mu\partial_\mu + \beta\partial_\lambda - \frac{\ell}{2}\gamma_{m^2} - \frac{n}{2}\gamma_\phi\right)\Gamma^{(n,\ell)}(p_i; \lambda, 0, \mu) = 0}$$

(4.157)

defined in the massless theory ($m^2=0$). This defines $(\beta, \gamma_{m^2}, \gamma_\phi)$ as $\beta(\lambda), \gamma_{m^2}(\lambda), \gamma_\phi(\lambda)$, being mass-independent. Such a RG-scheme is called

<u>mass-independent scheme</u>.

The solution of eq. (4.152) is obtained in the same spirit as eq. (4.140):

$$\boxed{\Gamma^{(n)}\big(p_i;\lambda(s),m^2(s),\mu(s)\big)=e^{\frac{n}{2}\int_{\lambda}^{\lambda(s)}\frac{\gamma_\phi(\lambda')}{\beta(\lambda')}\,d\lambda'}\cdot\Gamma^{(n)}\big(p_i;\lambda,m^2,\mu\big)}\tag{4.158}$$

with
$$\underbrace{s\frac{d}{ds}}_{\left(\overset{\mu\partial_\mu}{\overset{\shortparallel}{s\partial_s}}+\beta_\lambda\partial_\lambda-\gamma_{m^2}m^2\partial_{m^2}\right)}\Gamma^{(n)}\big(p_i;\lambda(s),m^2(s),\mu(s)\big)=\frac{n}{2}\gamma(\lambda(s))\,\Gamma^{(n)}.$$

Again we are most interested in the momentum behaviour. With the canonical scaling, eq. (4.142), p. 15? and eq. (4.158) we are led to

$$\boxed{\Gamma^{(n)}\big(s\,p_i;\lambda,m^2,\mu\big)=s^{d-n\left(\frac{d-2}{2}\right)}\,e^{-\frac{n}{2}\int_{\lambda}^{\lambda(s)}\frac{\gamma_\phi(\lambda')}{\beta(\lambda')}\,d\lambda'}\cdot\Gamma^{(n)}\big(p_i;\lambda(s),m^2(s),\mu\big)}\tag{4.156}$$

Note that we have chosen dimensionless couplings $\vec{\lambda} = (\lambda, m^2)$ with $\vec{\beta} = \mu \frac{d}{d\mu}(\lambda, m^2) = (\beta, \gamma_{m^2})$

The dimensionless couplings derive from the dimensionful ones with appropriate factors of μ (k in the case of the flow).

With these definitions we have

$$m^2(s) = m^2 \cdot e^{-\int_0^{\ln s} \gamma_{m^2}(s') \frac{ds'}{s'}}$$
$$\lambda(s) = \lambda \cdot e^{\int_0^{\ln s} \beta(s') \frac{ds'}{s'}} \qquad (4.157)$$

From the first eq. in eq. (4.157) we conclude that for $m^2(s) = 1$ we have

$$\boxed{m^2 = e^{\int_0^{\ln s} \gamma_{m^2}(s') \frac{ds'}{s'}}} \qquad (4.158)$$

With $m^2(s) = 1$ we arrange for $\Gamma^{(n)}(p_i, \lambda(s), m^2(s), \mu)$ being away from the critical region in eq. (4.156).

Now we tune $m^2 \to 0$, approaching the phase transition while keeping $m^2(s) = 1$.

$m^2 \to 0$ implies $s \to 0$ and hence

$$\lambda(s) \to \lambda_*, \quad \underbrace{\frac{m^2 \mu(s)^2}{\mu^2}}_{\bar{m}^2} \simeq e^{(2+\gamma_{m^2}(\lambda_*))\ln s} = s^{(2+\gamma_{m^2}(\lambda_*))} \tag{4.159}$$

Eq. (4.159) entails $s = (\bar{m}^2)^{\frac{1}{2+\gamma_{m^2}}}$ which suggests the identification $\nu = \frac{1}{2+\gamma_{m^2}}$. We write

$$\boxed{\begin{aligned}\Gamma^{(n)}(p_i;\lambda,m^2,\mu) &= (\bar{m}^2)^{\frac{1}{2+\gamma_{m^2}}\left(d - n\left[\frac{d-2}{2} - \gamma_\phi/2\right]\right)} \\ &\cdot \Gamma^{(n)}\left(\frac{p_i}{(\bar{m}^2)^{\frac{1}{1+\gamma_{m^2}}}};\lambda_*,1,\mu\right)\end{aligned}} \tag{4.160}$$

with a rescaling $p_i \to p_i/s$ in eq. (4.156).

For $n=2$, eq. (4.160) reads

$$\boxed{\Gamma^{(2)}(p;\lambda,m^2,\mu) = \mu^2 (\bar{m}^2)^{\frac{1}{2+\gamma_{m^2}}(2-\gamma_\phi)}\, \Gamma^{(2)}\left(\frac{p}{\mu}\left(\frac{1}{\bar{m}^2}\right)^{\frac{1}{2+\gamma_{m^2}}};\lambda_*,1,1\right)} \tag{4.161}$$

This has to be compared with the critical form of the propagator, eq. (1.76), p. 36,

$$\tilde{G}(p)^{-1} = (p^2)^{1-\eta/2} f^{-1}(\xi \cdot p) \quad (4.162)$$

with $f(x \to \infty) \to \text{const}$ and $f(x \to 0) \sim x^{2-\eta}$.

For $\bar{m}^2 = \left(\frac{\mu^2}{p^2}\right)^{1+\gamma_{m^2}} \to 0$ we have

$$\Gamma^{(2)}(p; \lambda, m^2, \mu) = p^{2(1-\gamma_\phi/2)} \Gamma^{(2)}(1, \lambda_\phi, 1, 1)$$

and we identify

$$\boxed{\eta = \gamma_\phi, \quad \nu = \frac{1}{2+\gamma_{m^2}} \quad (4.163)}$$

We also define the total scaling dimension of the field

$$\boxed{d_\phi = \frac{d-2}{2} + \eta/2 \qquad (4.164)}$$

Remarks:

(i) The results eq. (4.164) and eq. (4.154) have to be compared with the results from the FRG. While β_u and β_λ vanish at the FP, $\gamma_{m^2} = \frac{1}{\nu} - 2 \neq 0$. Note, however, that $\gamma_{m^2} = \mu\left(\frac{d}{d\mu} m^2\right)/m^2\Big|_{m^2=0}$ whereas $\beta_{u} = \partial_t u$ with $m^2 = 2\lambda u$ and hence:

$$\gamma_{m^2} \sim \beta_u/m^2\Big|_{m^2=0} = \frac{\partial \beta_u}{\partial m^2}\Big|_{m^2=0}, \quad (4.165)$$

where $\partial\beta_u/\partial m^2$ relates to an entry in the stability matrix B.

(ii) γ_{m^2} relates to the anomalous dimension at $m^2=0$ of the composite op. $[\phi^2]$, the related stability matrix is diagonal, hence eq. (4.164).