5.3 1/N Expansion

In the last chapter we have seen that we can solve the flow eq. analytically in the large N-limit. In that computation we have dropped the sub-leading $1/N$-term in eq. (5.58). The corresponding loop is that of the radial mode.

Here we utilize this ordering for expanding the $O(N)$-theory in orders of $1/N$.

As starting point we take the results for $N = \infty$. We recall the eq. for the β-fct. eq. (5.37), p. 178 in the rescaled variables, $A_d = \frac{1}{d} \frac{\Omega_d}{(2\pi)^d}$,

$$\beta_\lambda = (d-4)\lambda + 2 A_d \lambda^2 \left(1 + \frac{1}{N-1} \frac{9}{(1+2\lambda\kappa)^3}\right)$$

$$\beta_\kappa = (2-d)\kappa + A_d \lambda \left(1 + \frac{3}{(1+2\lambda\kappa)^2}\right) \qquad (5.65)$$

and hence in the limit $\frac{1}{N-1} \to 0$:

$$\beta_\lambda = (d-4)\lambda + 2A_d\lambda^2$$
$$\beta_\kappa = (2-d)\kappa + A_d\lambda \qquad (5.66)$$

with the non-Gaussian FP $(4>d>2)$

$$\lambda_* = \frac{4-d}{2A_d}, \quad \kappa_* = \frac{1}{2}\frac{4-d}{d-2} \qquad (5.67)$$

The stability matrix $B_{ij} = \left.\frac{\partial\beta_i}{\partial\lambda_j}\right|_{\vec{\lambda}_*}$, $\vec{\lambda} = (\kappa, \lambda)$, see p. 139
is given by

$$B = \begin{pmatrix} (2-d) & A_d \\ 0 & 4-d \end{pmatrix} \qquad (5.68)$$

with EV

$$\omega_0 = 2-d, \quad \omega_1 = 4-d$$

$$\Rightarrow \boxed{\nu = \frac{1}{d-2}} \qquad (5.69)$$

The other independent critical exponent η vanishes,

$$\boxed{\eta = 0} \qquad (5.70)$$