Critical Phenomena

1) Calculate the free energy for an Ising spin chain, where there is ferromagnetic nearest neighbour interaction ($J_1 > 0$) and antoferromagnetic next to nearest neighbour ($J_2 < 0$) interaction, using the transfer matrix method.

$$H = -J_1 \sum_{i=1}^{N} s_i s_{i+1} - J_2 \sum_{i=1}^{N} s_i s_{i+2} \tag{1}$$

2) The Ising model in a homogeneous magnetic field background is defined by the following Hamiltonian:

$$H = B \sum_{i=1}^{N} s_i - J \sum_{i=1}^{N} s_i s_{i+1} \tag{2}$$

We define the quantity:

$$E_i = -\frac{J}{2} \sum_{j=i\pm1} s_i s_j \tag{3}$$

The zero magnetic field Hamiltonian can thus be written as $H = \sum_i E_i$.

a) Use the transfer matrix method to calculate the partition function. Calculate the magnetization by using the B as a current.

b) Compute the four-point function, and its connected part, in zero magnetic field.

$$\langle S_i S_j S_k S_l \rangle, \quad i < j < k < l \tag{4}$$

c) Show that the specific energy per spin, $\hat{C} = c/N$ in zero magnetic field can be related to the connected correlation function $\langle E_i E_j \rangle_c$ (energy-energy correlation).

$$\hat{C} = k\beta^2 \sum_j \langle E_i E_j \rangle_c = k\beta^2 \sum_j \langle (E_i - \langle E_i \rangle)(E_j - \langle E_j \rangle) \rangle \tag{5}$$

d) Calculate $\langle E_i E_j \rangle_c$, and use the result to check with the heat capacity result of the first tutorial! (Hint: consider $j = i,\ j = i+1,\ j = i+p,\ p \geq 2$ separately!)