1) Renormalize the effective potential in three dimensions! The Hamiltonian is given by:

$$H[M] = \int d^3x \left(\frac{1}{2}(\partial_\mu M(x))^2 + \frac{1}{2} r_0 M(x)^2 + \frac{1}{4!} u_0 M(x)^4 + B(x)M(x) \right) \tag{1}$$

The renormalised, finite mass parameter is defined as

$$r = \left. \frac{\partial^2 \Gamma[M]}{\partial M^2} \right|_{M=0} \tag{2}$$

Show that

$$r = r_0 + \frac{u_0}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + r_0} + \text{higher order terms} \tag{3}$$

What does this imply for r_0? What does change in 4d? (Bonus question: Compute the renormalized masses & couplings!)