Critical Phenomena — 1 -

1) Suppose some system has a correlator given by:

$$G(\mathbf{p}) = \frac{1}{p^2 + m^2} \tag{1}$$

Calculate the correlator $G(\mathbf{x})$ in real space in 1,2 and 3 dimensions. What is the correlation length?

2) The Legendre transform of the function $f(x)$ is defined by

$$g(y) = \sup_x (xy - f(x)) \tag{2}$$

How can one construct the Legendre transform graphically? Do this for a double well potential potential $(H(x) = r_0 x^2 + u_0 x^4, \quad r_0 < 0)$. What is the Legendre transform of the first Legendre transform?

In the case of a Gaussian theory, the Hamiltonian is written as:

$$H = \int d^3x \left(\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2} r_0 \varphi^2 \right) \tag{3}$$

calculate $\Gamma[M]$ exactly!