Critical Phenomena - 1 -

1) Consider a fermionic lattice model with the following Hamiltonian:

$$H[\psi, \bar{\psi}] = \sum_{i,j} \bar{\psi}_j D_{ji} \psi_i \tag{1}$$

with a given Dirac operator D_{ij}. Using the linear blocking functional:

$$\psi_{f,i} = \sum_{ij} f_{ij} \psi_j, \tag{2}$$

calculate the generating functional $H'[\eta, \bar{\eta}]$ defined by:

$$e^{-H'[\eta, \bar{\eta}]} = \int D\psi D\bar{\psi} e^{-H[\psi, \bar{\psi}] - (\bar{\eta}_i - \bar{\psi}_{f,i}) A_{ij} (\eta_j - \psi_{f,j})} \tag{3}$$

with some smearing function A_{ij}.

Calculate the new Dirac operator $\hat{D}$, defined by

$$H'[\eta, \bar{\eta}] = \sum_{ij} \bar{\eta}_j \hat{D}_{ji} \eta_i \tag{4}$$

As shown in the lecture, $\{\gamma_5, D\} = 0$ implies $\{\gamma_5, \hat{D}\} = \hat{D}\{\gamma_5, A^{-1}\}\hat{D}$.

What happens after performing one more blocking?