

## Critical Phenomena

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### 1) Perturbative Functional Renormalization

Consider a scalar field theory with classical action

$$S = \frac{1}{2} \int d^4x \Phi(x) (-\nabla^2 + m^2) \Phi(x) + \frac{\lambda}{24} \Phi^4(x),$$

and calculate  $\Gamma_k^{1\text{-loop}}[\Phi]$ , the one-loop blocked effective action by integrating the flow equation!

Calculate the flow equation for  $\Gamma_k^{1\text{-loop}(2)}[\Phi]$  the second functional derivative of the effective action.

The running mass-parameter is defined as

$$\Gamma_k^{(2)}[\Phi] \Big|_{\Phi=0, p=0} = m_k^2$$

Calculate the running of the mass parameter, and show that it can be written as

$$\Lambda \partial_\Lambda m_\Lambda^2 = \text{const.} \Lambda^2,$$

in leading order in  $\Lambda$ . Using the cutoff function

$$R_k(p) = (k^2 - p^2) \Theta(k^2 - p^2),$$

calculate the running of the mass parameter explicitly!