Geometry & Topology in Physics

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Tuesday, 11:15-13:00, Pw 19 / SR [LSF]

Wednesday, 11:15-13:00, every 2nd week, kHS Pw 12 [LSF]

- Content
- Literature
- Exercises & bonus material
- Script

Prerequisites: Theoretical Physics I-IV, basic knowledge of QFT

Content of lecture series

The lecture course provides an introduction to geometrical and topological effects in physics, applications range from quantum mechanics to quantum field theory.

Outline in key words

- **Symmetries & topological exitations:**
  Scalar field theory, solitons, fermions & index theorems, dilute gas expansion

- **Gauge theories & homotopy:**
  Abelian Higgs model, Bogomol'nyi bound, homotopy, topological invariants & classification

- **Non-Abelian gauge theories:**
  Setting, instantons, zero modes, collective coordinates & moduli space, fermionic zero modes & Atiyah-Singer index theorem, topology & confinement

LINKS
Institute for Theoretical Physics

ExtreMe Matter Institute EMMI

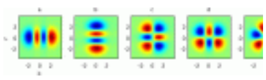
DFG research group FOR 723

Research Training Group Quantum Many-body Dynamics and Nonequilibrium Physics

Department of Physics and Astronomy

Graduate School of Fundamental Physics

Graduate Academy



- **Gauge anomalies:**
  Cocycles, Wess-Zumino consistency condition, consistent & covariant anomalies

- **Seiberg-Witten:**
  Supersymmetry, chiral multiplets, N=2 Susy Yang-Mills, super-conformal Ward-Id's, dualities

- **Topology & dynamics:**
  Dynamics, vortices, scaling

Literature

Coleman	Aspects of Symmetry	Cambridge University Press
Göckeler & Schücker	Differential Geometry, gauge theories, and gravity	Cambridge University Press
Nakahara	Geometry, Topology and Physics	Hilger
Nash & Sen	Topology and Geometry For Physicists	Academic
Rajaraman	Solitons and Instantons	North-Holland
Wu-Ki Tung	Group Theory in Physics	World Scientific
Zinn-Justin	Quantum Field Theory and Critical Phenomena	Oxford
	Lecture notes	
Bruckmann	Topological objects in QCD	Lecture notes, Schladming winter school 2007
Lenz	Topological concepts in gauge theories	Lecture notes
't Hooft, Bruckmann	Monopoles, Instantons and Confinement	Lecture notes, Saalburg summer school 1999

Motivation

EoM : $S[\phi]$ action, $S[\phi] = \int d^dx\, \mathcal{L}(\phi)$

field theory: $\phi = \phi(\vec{x}, t)$

mechanics: $\phi = \vec{q}(t)$

Physics: $$\left.\frac{\delta S}{\delta \phi}\right|_{\bar{\phi}} = 0$$

Set of $\bar{\phi}$ possible physics evolution uniquely determined by initial cond./ boundary conditions.

Quantum Physics:

EoM: $\Gamma[\phi]$ effective action

$$\left.\frac{\delta \Gamma}{\delta \phi}\right|_{\bar{\phi}} = 0$$

$\bar{\phi}$ mean field / classical field $\langle \phi \rangle$

$$\left.\frac{\delta^n \Gamma}{\delta \phi^n}\right|_{\bar{\phi}} \simeq \langle \phi^n \rangle$$

How to compute?

- First (simplest) approach:

Perturbation theory

(i) theory is basically free / classical

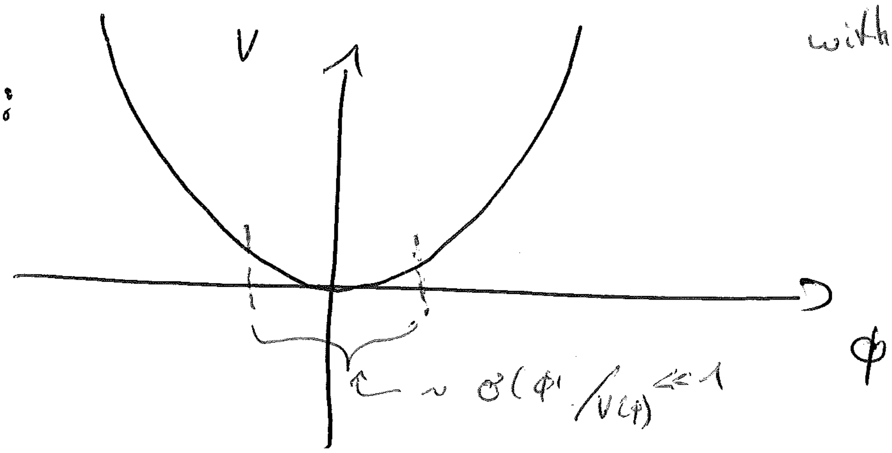
(ii) quantum fluctuations are perturbations

Example: real scalar theory in 1+1 dimensions

$$S[\phi] = -\frac{1}{2}\int d^2x\, \partial_\nu\phi\partial^\nu\phi \quad \int d^2x\, V(\phi)$$

with $V(\phi)=V(-\phi)$

$V(\phi)$:



$\bar{\phi} = 0$: $\quad V(\phi) = V(0) + \frac{1}{2} V''(0)\,\phi^2 + \frac{1}{4!} V^{(4)}(0)\,\phi^4 + \mathcal{O}(\phi^6)$

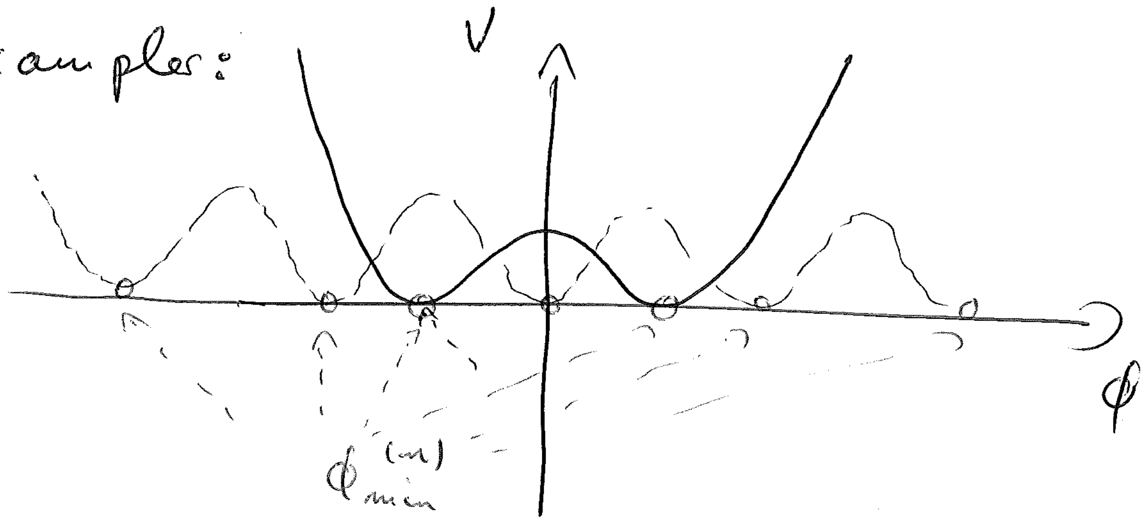
$V(\phi)=V(-\phi)$

Problems: (1) convergence (at most asymptotic series (with radius of convergence 0))

Non-perturbative (NP) domain {
(2) strong coupling (evidently)
(3) topological effects

(3) is the topic of this lecture

Examples:



- $\phi(t \to \pm\infty) \to \phi^{(n)}_{min}$ for finite Energy solutions

- $(\phi(t=-\infty), \phi(t=+\infty))$ cannot be changed smoothly into $(\phi'(t=-\infty), \phi'(t=+\infty)) \neq (\phi(t=-\infty), \phi(t=+\infty))$

in general: $\phi : \mathcal{M}_{\text{space-time}} \to \mathcal{M}_{\text{Target space}}$

- Quantisation within saddle point expansion about <u>all</u> <u>distinct</u> vacua / classical solutions $\sim e^{iS[\phi_n]/\hbar}$
  $(\neq \sum c_m \hbar^m)$

$\Rightarrow$ (i) classification of solutions mandatory, construction of solutions ...

(ii) classical solutions carry interesting non-perturbative physics

(iii) disclaimer: beware of naive belief of topological arguments (key word: Instanton behind the moon argument)

in short: Topology is global, (most) physics is local

$\Rightarrow$ topological densities important

Applications

- quantum mechanics (tunneling, geometric phases)
- solid states physics (vortices, tuneling..)
- QFT / elementary part. phys. (anom., chiral sym. breaking, confinement, electro-weak phase trans., cosmology; topo field theories, SW, gauge fixing (global), ...)
- string theory (dualities, fluxes, instantons, ...) (an exercise in algebraic geometry)