Geometry and Topology in Physics

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Content of lecture series

The lecture course provides an introduction to geometrical and topological effects in physics, applications range from quantum mechanics to quantum field theory.

Keywords

- Symmetries
- Topological exitations
- Anomalies
- Semi-classical considerations
- Applications
  - Abelian/Non-Abelian gauge theories
  - Supersymmetry
  - Confinement & anomalous chiral symmetry breaking

Literature

Coleman	Aspects of Symmetry	Cambridge University Press
Göckeler & Schücker	Differential Geometry, gauge theories, and gravity	Cambridge University Press
Nakahara	Geometry, Topology and Physics	Hilger
Nash & Sen	Topology And Geometry For Physicists	Academic
Rajaraman	Solitons And Instantons	North-Holland
Wu-Ki Tung	Group Theory in Physics	World Scientific
Zinn-Justin	Quantum Field Theory and Critical Phenomena	Oxford

Lecture notes

Bruckmann	Topological objects in QCD	Lecture notes, Schladming winter school 2007
Lenz	Topological concepts in gauge theories	Lecture notes
't Hooft	Monopoles, Instantons and Confinement	Lecture notes, Saalburg summer school 1999

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Kommentare per E-Mail an J.M. Pawlowski

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Motivation

EoM : $S[\phi]$ action, $S[\phi] = \int d^dx \, \mathcal{L}(\phi)$

field theory: $\phi = \phi(\vec{x}, t)$

mechanics: $\phi = \vec{q}(t)$

Physics: $$\left.\frac{\delta S}{\delta \phi}\right|_{\bar{\phi}} = 0$$

Set of $\bar{\phi}$ possible physics evolution uniquely determined by initial cond./ boundary conditions.

Quantum Physics:

EoM: $\Gamma[\phi]$ effective action

$$\left.\frac{\delta \Gamma}{\delta \phi}\right|_{\bar{\phi}} = 0$$

$\bar{\phi}$ mean field / classical field $\langle \phi \rangle$

$$\left.\frac{\delta^n \Gamma}{\delta \phi^n}\right|_{\bar{\Phi}} \simeq \langle \phi^n \rangle$$

How to compute?

- First (simplest) approach:

Perturbation theory

(i) theory is basically free / classical

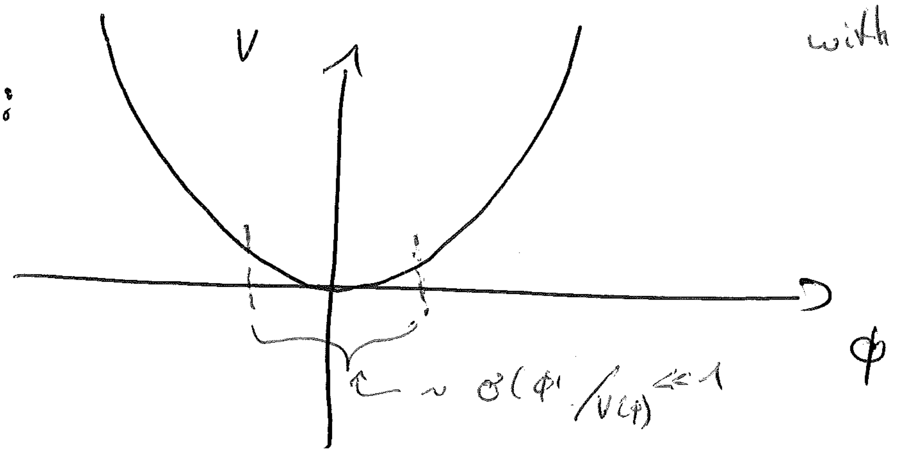
(ii) quantum fluctuations are perturbations

Example: real scalar theory in 1+1 dimensions

$$S[\phi] = -\frac{1}{2}\int d^2x\, \partial_\nu \phi \partial^\nu \phi \quad \int d^2x\, V(\phi)$$

with $V(\phi) = V(-\phi)$

$V(\phi)$:



$\bar{\Phi} = 0$:

$$V(\phi) = V(0) + \frac{1}{2} V''(0)\, \phi^2 + \frac{1}{4!} V^{(4)}(0)\, \phi^4 + \mathcal{O}(\phi^6)$$

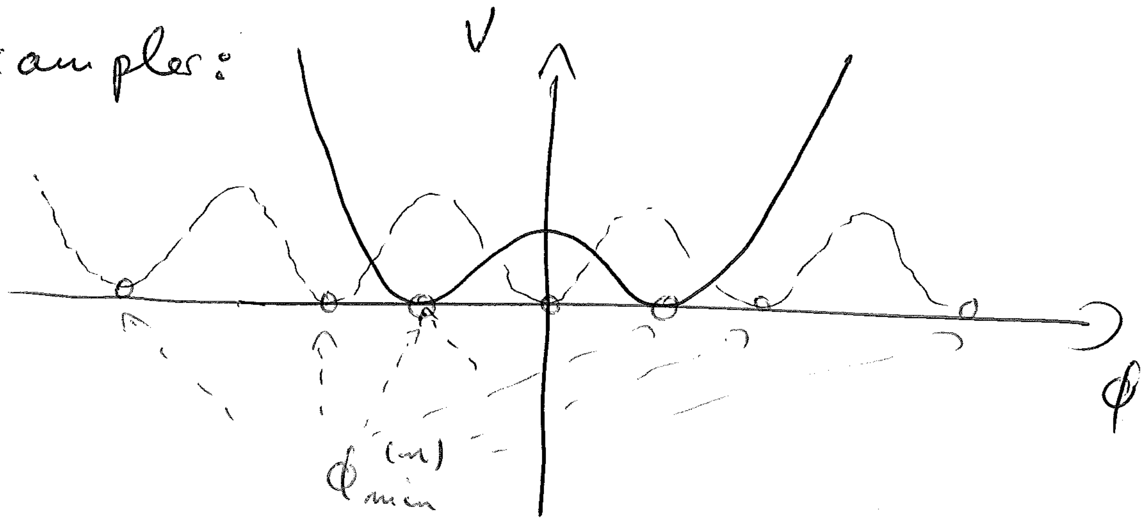
$V(\phi) = V(-\phi)$

Problems: (1) convergence (at most asymptotic series (with radius of convergence 0))

Non-perturbative (NP) domain:
- (2) strong coupling (evidently)
- (3) topological effects

(3) is the topic of this lecture

Examples:



- $\phi(t \to \pm\infty) \to \phi^{(n)}_{min}$ for finite Energy solutions

- $(\phi(t=-\infty), \phi(t=+\infty))$ canot be changed smoothly into $(\phi'(t=-\infty), \phi'(t=+\infty)) \neq (\phi(t=-\infty), \phi(t=+\infty))$

in general: $\phi : \mathcal{M}_{\text{Spacetime}} \to \mathcal{M}_{\text{Target space}}$

- Quantisation within saddle point expansion about <u>all</u> <u>distinct</u> vacua / classical solutions $\sim e^{iS[\phi_n]/\hbar}$
  $\left(\neq \sum c_m \hbar^m\right)$

$\Rightarrow$ (i) classification of solutions mandatory, construction of solutions ...

(ii) classical solutions carry interesting non-perturbative physics

(iii) disclaimer: beware of naive belief of topological arguments (key word: Instanton behind the moon argument)

in short: Topology is global, (most) physics is local

$\Rightarrow$ topological densities important

Applications

- quantum mechanics (tunneling, geometric phases)
- solid states physics (vortices, tunneling..)
- QFT/elementary part. phys. (anom., chiral sym. breaking, confinement, electro-weak phase trans., cosmology; topo field theories, SW, gauge fixing (global), ...)
- string theory (dualities, fluxes, instantons, ...)
  (an exercise in algebraic geometry)