3.4 Gauge Anomalies (a glimpse)

Consider a chiral gauge theory

$$S[A,\psi,\bar\psi] = S_{YM}[A] + S_W[A,\psi,\bar\psi]$$

with

$$S_W[A,\psi,\bar\psi] = i\int d^4x\, \bar\psi\, \gamma_\nu(\partial_\nu + P_- A_\nu)\psi$$

$$P_\pm = \frac{1\pm\gamma_5}{2} \quad \text{chiral projection operators}$$

1 left-handed interacting Weyl fermion
1 right-handed free " "

Gauge symmetry: $D_\nu = \partial_\nu + A_\nu$

$$A_\nu^{\mathcal{U}} = \mathcal{U} D_\nu \mathcal{U}^\dagger$$

$$\psi^{\mathcal{U}} = (\mathbb{1} + P_- \mathcal{U})\psi$$

$$\bar\psi^{\mathcal{U}} = \bar\psi(\mathbb{1} + P_+ \mathcal{U}^\dagger)$$

Important for
- SM: electroweak sector
- susy: Weyl fermions

Gauge current: $u = e^{\alpha^a t^a}$

$$D_\nu J^a_\nu = i\frac{\delta S_W}{\delta \alpha^a} = i\underbrace{\left(\bar{\Psi} t^a \gamma_\nu P_- \Psi\right)}_{J^a_\nu}$$

As for the Abelian anomaly

$$Z_W[\eta, \bar{\eta}; A] = \int D\Psi D\bar{\Psi} e^{-S_W[A,\Psi,\bar{\Psi}] + \int d^4x(\bar{\eta}\Psi - \bar{\Psi}\eta)}$$

infinitesimally: $\left.\frac{\delta Z_W[0,0;A]}{\delta \alpha}\right|_{\alpha=0} = 0$

$$\boxed{i D_\nu \langle \bar{\Psi} t^a \gamma_\nu P_- \Psi \rangle = i \underbrace{\frac{\delta J}{\delta \alpha^a}}_{A_{cov}}}$$

covariant anomaly (if reg is inserted at the end)

or

$$\boxed{D^{ab}_\nu \frac{\delta}{\delta A^b_\nu} \ln Z_W[0,0;A] = A^a}$$

consistent anomaly

covariant anomaly:

$$\mathcal{A}^a_{cov} = i \frac{\delta J}{\delta \alpha^a}\Bigg|_{\text{Heat kernel}} = \frac{1}{32\pi^2}\, \varepsilon_{\mu\nu\rho\sigma}\, \mathrm{tr}\, t^a F_{\mu\nu} F_{\rho\sigma}$$

is derived from covariant regularisation of Jacobian J.

consistent anomaly:

$$\mathcal{A}^a = \frac{1}{24\pi^2}\, \varepsilon_{\mu\nu\rho\sigma}\, \partial_\mu \mathrm{tr}\, t^a \left(A_\nu \partial_\rho A_\sigma + \tfrac{1}{2} A_\nu A_\rho A_\sigma\right)$$

$$= \mathcal{A}^a_{cov} + D^{ab}_\mu \,\Delta J^b_\mu$$

with Bardeen-Zumino polynomial

$$\boxed{\Delta J^a_\mu = -\frac{1}{48\pi^2}\, \varepsilon_{\mu\nu\rho\sigma}\, \mathrm{tr}\, t^a \left(\{A_\nu, F_{\rho\sigma}\} - A_\nu A_\rho A_\sigma\right)}$$

computation: take A-dep. of regularisation into account!

Wess-Zumino consistency condition:

Due to Anomaly: $W[A] = \ln Z_W[0,0;A]$

is not gauge invariant

Group action:

usually: $\mathcal{R}(\mathcal{U})\,\Psi[A] = \Psi[A^{\mathcal{U}}]$

allow phase: $\mathcal{R}(\mathcal{U})\,\Psi[A] = e^{2\pi i \omega_1(A;\mathcal{U})}\,\Psi[A^{\mathcal{U}}]$

1-co-chain

compatibility with group multiplication

$$\mathcal{U}_1 \cdot \mathcal{U}_2 = \mathcal{U}_{12}$$

$$\mathcal{R}(\mathcal{U}_1)\circ\mathcal{R}(\mathcal{U}_2) = \mathcal{R}(\mathcal{U}_{12})$$

$$\Rightarrow \boxed{\Delta\omega_1 := \omega_1(A^{\mathcal{U}_1};\mathcal{U}_2) - \omega_1(A;\mathcal{U}_{12}) + \omega_1(A,\mathcal{U}_1) = 0 \mod n \in \mathbb{Z}}$$

cocycle condition

Δ: co-boundary operator

Anomaly: $\mathcal{A} = 2\pi \left.\frac{\delta\omega_1}{\delta\alpha}\right|_{\alpha=0}$

from

$$\mathcal{R}(\mathcal{U}_1)\circ\mathcal{R}(\mathcal{U}_2)\,\Psi[A] = \mathcal{R}(\mathcal{U}_1)\, e^{2\pi i\,\omega_1(A,\mathcal{U}_2)}\,\Psi[A^{\mathcal{U}_2}]$$

$$= e^{2\pi i\left(\omega_1(A,\mathcal{U}_2)+\omega_1(A^{\mathcal{U}_2},\mathcal{U}_1)\right)}\,\Psi[A^{\mathcal{U}_1\mathcal{U}_2}]$$

$$= \mathcal{R}(\mathcal{U}_1\circ\mathcal{U}_2)\,\Psi[A] = e^{2\pi i\,\omega_1(A,\mathcal{U}_{12})}\,\Psi[A^{\mathcal{U}_{12}}]$$

Russian formula for dimensional descent

$$\boxed{\Delta\,\omega_d = d\,\Omega_{d-1}}$$