4 Seiberg - Witten theory

$\mathcal{N} = 2$ supersymmetric Yang - Mills

Low energy effective action (Wilsonian effective action)

- perturbation theory 1-loop exact
- instanton contributions

$\Rightarrow$ full dynamics

Supersymmetry:

Coleman - Mandula:

- Symmetry group of QFT is isomorph to direct product of Poincaré and internal compact Liegroup

O' Raifeartaigh:

internal symmetries (Liegroups) cannot connect mass-shells

Exit: Supersymmetry: Graded Lie group: $[A,B] \to \{A,B\}$

Remark: O' Raifeartaigh theorem still intact

4.1 Supersymmetry

Extend Poincare with fermionic (Grassmann)

generators Q :

$$[A, B\} = AB - (-1)^{ab} BA$$

a, b : # of fermionic generators Q in A, B

Spin-statistics: Q carries spin

$$Q: \quad \underbrace{Q_\alpha^{\ i}, \quad \bar{Q}_{\dot\alpha}^{\ j}}_{\text{spin } 1/2}$$

$i, j = 1, \ldots, N$

N : # of generators

Details with consistency of algebra

$$[[A, B\}, C\} + \text{cyclic permuts } (-1)^{a(b+c)} = 0$$

$N = 1$ algebra: P: momentum, M: Lorentz group

$$\{ Q_\alpha, \bar{Q}_{\dot\beta} \} = 2 \sigma_{\alpha\dot\beta}{}^{\mu} P_\mu \leftarrow [Q] = 1/2$$

$([P] = 1)$

$$\{ Q_\alpha, Q_\beta \} = \{ \bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta} \} = 0$$

$$[Q_\alpha, P_\mu] = 0$$

$$[Q_\alpha, M_{\mu\nu}] = i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$[\bar{Q}^{\dot\alpha}, M_{\mu\nu}] = i(\bar\sigma_{\mu\nu})^{\dot\alpha}{}_{\dot\beta} \bar{Q}^{\dot\beta}$$

Poincaré

R-symmetry:

$$[Q_\alpha, R] = - Q_\alpha$$

chiral rotation

$$[\bar{Q}^{\dot\alpha}, R] = \bar{Q}^{\dot\alpha}$$

no central charge

Fields: ξ Grassmann, $\Phi = (\varphi, \psi, \dots)$

$$\delta_\xi \Phi = (\underbrace{\xi \cdot Q}_{\xi^\alpha Q_\alpha} + \underbrace{\bar\xi \bar Q}_{\bar\xi_{\dot\alpha} \bar Q^{\dot\alpha}}) \cdot \Phi$$

Start with scalar field: φ

$$\boxed{\delta_\xi \varphi =: \sqrt{2}\, \xi \cdot \psi} \leftarrow \text{fermion}$$

$$\Rightarrow \boxed{\delta_\xi \psi = \underbrace{i\sqrt{2}\, \sigma^\nu}_{\text{from } [\delta_\xi, \delta_\eta]\varphi} \cdot \bar\xi\, \partial_\nu \varphi + \sqrt{2}\, \xi \cdot F}$$

Tensor field

Also

$$\delta_\eta \delta_\xi \psi \doteq \boxed{\delta_\xi F = i\sqrt{2}\, \bar\xi \cdot \bar\sigma^\nu \partial_\nu \psi}$$

Fierz-identities

Analogously for

$$\varphi^*, \bar\psi, \bar F$$

$$\delta_\xi \varphi^* = \sqrt{2}\, \bar\xi \cdot \bar\psi$$

$$\delta_\xi \bar\psi = -i\sqrt{2}\, \xi \cdot \sigma^\nu \partial_\nu \varphi^* + \sqrt{2}\, \bar\xi F^*$$

$$\delta_\xi F^* = -i\sqrt{2}\, \partial_\nu \bar\psi \bar\sigma^\nu \cdot \xi$$

Invariant Lagrangian:

$$\mathcal{L}_0 = i(\partial_\nu \bar\psi)\bar\sigma^\nu \psi + \varphi^* \Box \varphi + F^* F + \underbrace{\varphi F + \varphi^* F^* - \tfrac{1}{2}\psi\cdot\psi - \tfrac{1}{2}\bar\psi\cdot\bar\psi}_{\text{mass terms}}$$

dofs:

bosonic	$\mathrm{Re}\,\varphi$	$\mathrm{Im}\,\varphi$	$\mathrm{Re}\,F$	$\mathrm{Im}\,F$
fermionic	ψ_1	ψ_2	$\bar\psi_{\dot 1}$	$\bar\psi_{\dot 2}$

4+4

Superspace: $z = (x_\nu, \theta_\alpha, \bar\theta_{\dot\alpha})$

for Realisation of susy-algebra

$$\mathcal{U}(x,\xi,\bar\xi) = e^{i\{x^\nu p_\nu + \xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}\}}$$

super field: (bosonic)

$$\phi(z) = \phi(x,\theta,\bar\theta)$$

$$\xi^\alpha Q_\alpha = \xi^\alpha\left(\frac{\partial}{\partial\theta^\alpha} - i\,\sigma_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial x^\nu}\right)$$

$$\bar\xi_{\dot\alpha}\bar Q^{\dot\alpha} = \xi_{\dot\alpha}\left(\frac{\partial}{\partial\bar\theta\alpha} - i\,\theta^\alpha\sigma_{\alpha\dot\beta}{}^\nu\varepsilon^{\dot\beta\dot\alpha}\frac{\partial}{\partial x^\nu}\right)$$

satisfy Susy-Alg

$$\delta\Phi(z) = \phi\left(x - a - i(\theta\sigma\bar\xi - \xi\sigma\cdot\bar\theta), \theta-\xi, \bar\theta-\bar\xi\right) - \phi(z)$$

Susy: $\mathcal{U}(a,\xi,\bar\xi)$

$$= 0$$

It follows, $\Theta^2 = \Theta^\alpha \Theta_\alpha$, $\bar\Theta^2 = \bar\Theta_{\dot\alpha} \bar\Theta^{\dot\alpha}$, metric $\varepsilon^{\alpha\beta}$

$$\Phi(x,\Theta,\bar\Theta) = C(x) + \Theta^\alpha \chi_\alpha + \bar\Theta_{\dot\alpha} \bar\eta^{\dot\alpha}$$
$$+ \Theta^2 M(x) + \bar\Theta^2 N(x) + \Theta^\alpha \bar\Theta^{\dot\alpha} V_{\alpha\dot\alpha}$$
$$+ \bar\Theta^2 \Theta^\alpha \lambda_\alpha + \Theta^2 \bar\Theta_{\dot\alpha} \bar\zeta^{\dot\alpha}(x)$$
$$+ \Theta^2 \bar\Theta^2 D(x)$$

chiral multiplet:

$$\bar D_{\dot\alpha} \Phi = 0$$

with

$$\bar D_{\dot\alpha} = \frac{\partial}{\partial \bar\Theta^{\dot\alpha}} - i \Theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \frac{\partial}{\partial x^\mu}$$
$$D_\alpha = \left(\frac{\partial}{\partial \Theta^\alpha} + i \sigma^\mu_{\alpha\dot\alpha} \bar\Theta^{\dot\alpha} \frac{\partial}{\partial x^\mu} \right)$$

$$\Rightarrow \quad \phi = \phi(y)$$

with $\quad y = x + i \Theta \sigma \bar\Theta$

Lagrangian:

$$\mathcal{L} = \int d^4\Theta\, \underbrace{K(\Phi, \Phi^+)}_{\text{Kähler pot.}} + \left(\int d^2\Theta\, \underbrace{W(\phi)}_{\text{superpot.}} + h.c. \right)$$

susy gen.:

$\boxed{a = \mu}$

$$\boxed{\delta_\xi = -\left(\xi^\alpha D_\alpha + \xi_{\dot\alpha} \bar{D}^{\dot\alpha} + \xi^a \partial_a\right)}$$

$[\delta, D_a] = 0 \quad \rightsquigarrow \quad D_a \xi^A = 0 \qquad A = (\alpha, \dot\alpha, a)$

$\rightsquigarrow$ concentrate on $\xi^a = 0$, as $\{D_\alpha, \bar{D}_{\dot\alpha}\}$ generates ∂_a!

$$Q_\alpha = \left(\underbrace{\frac{\partial}{\partial\theta^\alpha}}_{=\partial_\alpha} - i\sigma^a{}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_a\right)$$

$$\bar{Q}^{\dot\alpha} = \left(\frac{\partial}{\partial\bar\theta_{\dot\alpha}} - i\theta^\alpha\,\sigma^a{}_{\alpha\dot\beta}\,\varepsilon^{\dot\beta\dot\alpha}\partial_a\right)$$

$$\rightsquigarrow \bar{Q}_{\dot\alpha} = \left(\underbrace{-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}}_{=\partial_{\dot\alpha}} + i\theta^\alpha\sigma^a{}_{\alpha\dot\alpha}\,\partial_a\right)$$

$$D_\alpha = \left(\partial_\alpha + i\sigma^a{}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_a\right)$$

$$\bar{D}_{\dot\alpha} = \left(\partial_{\dot\alpha} - i\theta^\alpha\,\sigma^a{}_{\alpha\dot\alpha}\,\partial_a\right)$$