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Exercise 4

1) *Two vortices in a superfluid*

a) Consider two parallel vortices of strengths n_1 and n_2 separated by a distance d in a cylindrical container of radius R. The vortices move in a uniform medium of density ϱ and are well-separated from each other and from the boundary. Show that the interaction energy per unit length is given by

$$\mathcal{E}_{int} = \frac{\varrho}{2} \int d^2x \, \vec{v}_1 \cdot \vec{v}_2 = 4\pi n_1 n_2 \varrho \ln \frac{R}{d} \, , \tag{1}$$

up to logarithmic corrections.

b) Calculate the energy of a pair of $n_{1,2} = \pm 1$ vortices with separation d. Show that one can define an effective momentum for the vortex pair.

1) *Two-dimensional point-vortex gas*

a) Now, consider N parallel vortices of strengths n_i, $i = 1, \ldots, N$ in an infinite uniform medium. Construct the Hamiltonian for the N-point vortex system and derive the equations of motion.

b) Calculate the temperature of the N point-vortex gas in the microcanonical ensemble. What happens in the different temperature regimes?