Quantum Field Theory I

Content of lecture series

In the lecture course an introduction to quantum field theory is given.

Outline

- Classical field theory:
  space-time symmetries, internal symmetries, Noether theorem, conservation laws
- Quantisation:
  quantisation of free fields: bosons & fermions, Wick's theorem, Scattering matrix, Green functions
- Perturbation theory:
  Feynman rules, renormalisation, radiative corrections, LSZ, path integral
- Applications:
  QED, Non-Abelian gauge theories, Spontaneous symmetry breaking, standard model

Literature

- ***Quantum field theory, basics***

Haag	Local Quantum Physics	Springer, 1996
Itzykson, Zuber	Quantum Field Theory	McGraw-Hill, 1980
Mandl, Shaw	Quantum Field Theory	Wiley, 1993
Peskin, Schroeder	An Introduction to Quantum Field Theory	Addison Wesley, 1995
Ramond	Field Theory. A Modern Primer	Addison Wesley, 1999
Ryder	Quantum Field Theory	Cambridge UP, 1996

LINKS
Institute for Theoretical Physics

ExtreMe Matter Institute EMMI

DFG research group FOR 723

Research Training Group Simulational methods in Physics

Department of Physics and Astronomy

Graduate School of Fundamental Physics

Graduate Academy

Siegel	Fields	hep-th/9912205
Srednicki	Quantum Field Theory	Cambridge UP, 2007
Stone	The Physics of Quantum Fields	Springer, 2000
Weinberg	The Quantum Theory of Fields, Vol. 1-2	Cambridge UP, 1996

- ***Quantum field theory, applications***

Kugo	Eichtheorie	Springer, 1997
Miransky	Dynamical Symmetry Breaking in Quantum Field Theories	World Scientific, 1993
Muta	Foundations of Quantum Chromodynamics	World Scientific, 1987
Nachtmann	Elementarteilchenphysik - Phänomene und Konzepte	Vieweg,1992
Pokorski	Gauge Field Theories	Cambridge UP, 1987
Wu-Ki Tung	Group Theory in Physics	World Scientific, 1985
Zinn-Justin	Quantum Field Theory and Critical Phenomena	Oxford UP, 1993

- ***Textbooks on the renormalisation group and critical phenomena***

Amit	Field Theory, the Renormalization Group, and Critical Phenomena	World Scientific
Binney, Dowrick, Fisher, Newman	The Theory of Critical Phenomena, an Introduction to the Renormalization Group	Clarendon Press, Oxford
Cardy	Scaling and Renormalization in Statistical Physics	Cambridge UP
Collins	Renormalization	Springer
Parisi	Statistical Field Theory	Addison-Wesley

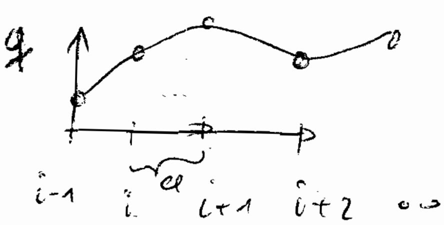
Quantum Field Theory

1 Introduction

Quantum Field Theory describes the fundamental interactions of matter.

$$\begin{array}{ccc} x, p & & \phi(\vec{x}),\ \pi(\vec{x}) \\ \text{classical mechanics} & \longrightarrow & \text{classical field theory} \\ [\hat{x},\hat{p}] = i\hbar \ \Big\downarrow & & [\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x})] = i\hbar \, \delta(\vec{x}-\vec{y}) \ \Big\downarrow \\ \text{quantum mechanics} & \longrightarrow & \text{quantum field theory} \\ \text{operators } \hat{x}, \hat{p} & & \text{operators } \hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}) \end{array}$$

Example I: oscillating string; mass



$$\xrightarrow{a \to 0} \phi(x,t)$$

$$\partial_t^2 q_i = -c^2 (q_i - q_{i-1} + q_i - q_{i+1}) / a^2 \qquad (1.1)$$

$$\downarrow$$

$$\partial_t^2 \phi(t,x) = -c^2 \, \partial_x^2 \phi(t,x) \longleftarrow \left[\frac{q_i - q_{i-1}}{a} \to \partial_x \phi \right]$$

$$S[q] = \int dt \, \frac{1}{2} \sum_i a \left\{ (\partial_t q_i)^2 - c^2 (q_{i+1} - q_i)^2 / a^2 \right\}$$

$$\Big\downarrow a \to 0 \qquad (1.2)$$

$$S[\phi] = \int dt \int dx \left\{ (\partial_t \phi)^2 - c^2 (\partial_x \phi)^2 \right\}$$

in general dimensions: (with $c=1$)

$$S[\phi] = \int d^d x \left\{ \frac{1}{2} (\partial_t \phi)^2 - (\vec{\nabla} \phi)^2 - V(\phi) \right\} \qquad (1.3)$$

(i) 'simply a bunch of (coupled) harmonic oscillators'

(ii) $S[\phi]$ has Poincaré invariance → later

Example II: Electrodynamics

action: $$S[A_\nu] = \int d^4x\, \mathcal{L}(A_\nu(x), \partial_\mu A_\nu(x)) \tag{1.4}$$

$$\nu = 0, \underbrace{1,2,3}_{i} \quad , x^0 = t, \ (x^i) = \vec{x}$$
$$i = 1,2,3$$

Lagrangian: $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.5}$$

$$F^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma} F_{\rho\sigma}$$

with flat metric $$\eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}^{\mu\nu} \tag{1.6}$$

$$\eta_\mu{}^\nu = \delta_\mu{}^\nu \quad (= \eta^{\nu\rho}\eta_{\mu\rho})$$

Remarks: (i) $\eta^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\mu\nu}$ also used (mostly in GR, QG)

(ii) in general $$g^{\nu\rho}\, g_{\mu\rho} = \delta_\mu{}^\nu \tag{1.7}$$

inverse metric — metric

(iii) Content of lecture course see cover sheet

In both examples

$$\begin{array}{lll} q \to \phi, A & \xrightarrow{\text{Quantisation}} & \hat{\phi}, \hat{A} \\ p \to \underset{\overset{\shortparallel}{\pi_\phi}}{\dot{\phi}}, \underset{\overset{\shortparallel}{\pi_A}}{\dot{A}} & & \hat{\pi}_\phi, \hat{\pi}_A \end{array}$$

describes annihilatio and creation of particles

(1) Hilbertspace construction, related to operators, e.g., $\hat{\phi}$, $\hat{\pi}_\phi$.

vacuum $|\Omega\rangle$ annihilation
1 particle $|1\rangle$ creation
$\vdots$

(2) Modern particle physics described by (renormalisable) quantum field theories:
scalar fields (Higgs)
fermion fields: (leptons, quarks)
vector fields: (photons; $W^\pm, Z$; gluons)
graviton (spin 2)
(perturbatively) non-renormalisable

2 Free scalar field

2.1 Classical theory

(1) real scalar field $\phi(x)$ — fundamental field Higgs

composite field, e.g. meson

pions, ...

more precisely: $\phi(x)$ is a scalar

under Poincaré transformation

(Translations, Rotations, Boosts)

(+ discrete C, P, T)

$$\phi'(x') = \phi(x) \tag{2.1}$$

(2) action should be Lorentz invariant

relevant example: $S[\phi] = \int d^4x \, \mathcal{L}(\phi(x), \partial_\nu \phi(x))$

$$\mathcal{L} = \frac{1}{2} \partial_\nu \phi \, \partial^\nu \phi - V(\phi) \tag{2.2}$$

with $V(\phi) = \frac{1}{2} m \phi^2 + \mathcal{O}(\phi^3)$

constant term: irrelevant at $T = 0$

linear term: canceled by shift in ϕ

Lorentz trafos: $x_\mu \to \Lambda_\mu{}^\nu x_\nu$

with $\Lambda^T \cdot \eta \cdot \Lambda = \eta$

$$\Rightarrow \partial_\mu \phi \to \Lambda_\mu{}^\nu \partial_\nu \phi \qquad (2.3)$$

Invariance of $\mathcal{L}$:

$$\partial_\mu \phi \, \partial^\mu \phi \quad \partial_\nu \phi \underbrace{\Lambda_\mu{}^\nu \Lambda^\mu{}_\rho}_{(\Lambda^T \eta \Lambda)^\nu{}_\rho} \partial^\rho \phi$$

$$= \partial_\nu \phi \, \partial^\nu \phi$$

$$V(\phi) \to V(\phi) \qquad (2.4)$$

(3) Equation of motion (EoM): $\partial_\mu \phi \partial^\mu \phi = (\partial\phi)^2$

$$\delta S = 0 = \delta \int d^4x \left\{ \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right\}$$

$\delta(\partial\phi)^2 = (\delta\, \partial_\mu \phi \, \partial_\nu \phi)\eta^{\mu\nu} \to$
$$= \int d^4x \left\{ \partial_\mu \phi \left(\partial_\nu \delta\phi\right) \eta^{\mu\nu} - m^2 \phi \, \delta\phi \right\}$$

partial int. $\to$
$$= -\int d^4x \left\{ \eta^{\mu\nu} \partial_\mu \partial_\nu \phi + m^2 \phi \right\} \delta\phi$$

$$= -\int d^4x \, \delta\phi \left(\partial^2 + m^2\right) \phi \qquad (2.5)$$

$$\Rightarrow \boxed{(\partial^2 + m^2)\, \phi(x) = 0}$$ Klein-Gordon equation (2.6)