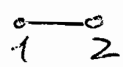
Then

$$T\,\phi_1 \cdots \phi_{n+1} = :\phi_1 \cdots \phi_{n+1} + \text{all contr.}:$$

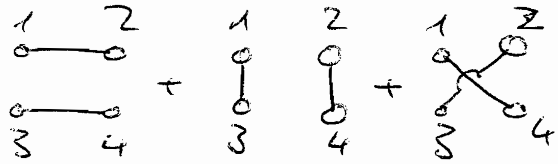
∎

3.3 Feynman rules

With Wick's theorem we write every time-ordered n-point fct. as a product of Feynman propagators. We introduce the diagrammatical notation

$$D_F(x_1-x_2) = \langle 0|T\,\phi_1\,\phi_2|0\rangle =$$ 

It follows e.g.:

$$\langle 0|T\,\phi_1 \cdots \phi_4|0\rangle =$$ 

What about $\langle 0|T\,\phi_1\,\phi_1|0\rangle$?

$$D_F(0) =$$ 

$D_F(x)$ was given on p. 56, eq. (3.49):

$$\sim \quad D_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon} \qquad 3.59$$

singular. As already done before, this singularity will be removed by an appropriate adjustment of our computation (renormalisation).

Remark: $\left[\int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} \right] = 2$

$$\Rightarrow D_F(0) = M^2 + \text{infinite}$$

Relevant example: 2-2 scattering:

(i) $O(\lambda^0)$: $\langle 0 | T \phi_1 \cdots \phi_4 | 0 \rangle$

(ii) $O(\lambda)$: $\phi = \phi(x)$

$$\frac{-i\lambda}{4!}\int d^4x \langle 0|\phi_1 \cdots \phi_4 \; \phi\phi\phi\phi|0\rangle$$

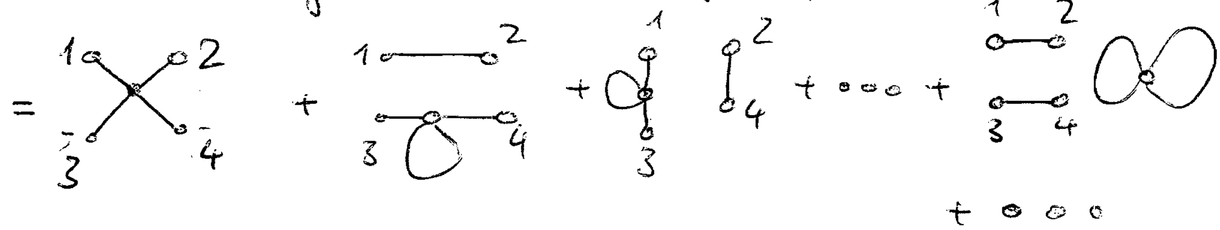
pos. to distr. ϕ^4 to $\phi_1, \phi_2, \phi_3, \phi_4$

$$= -\frac{i\lambda}{4!}\int d^4x \Big\{ \overline{\phi_1\phi}\;\overline{\phi_2\phi}\;\overline{\phi_3\phi}\;\overline{\phi_4\phi} \cdot 4! \qquad (3.60)$$

$$+ \overline{\phi\phi}\,\Big\{\overline{\phi_1\phi}\;\overline{\phi_2\phi}\;\overline{\phi_3\phi_4} + \text{perm.}\Big\}\cdot 12$$

$$+ \overline{\phi\phi}\;\overline{\phi\phi}\,\Big\{\overline{\phi_1\phi_2}\;\overline{\phi_3\phi_4} + \text{perm.}\Big\}\cdot 3$$

Diagrammatically (without sym. facts)

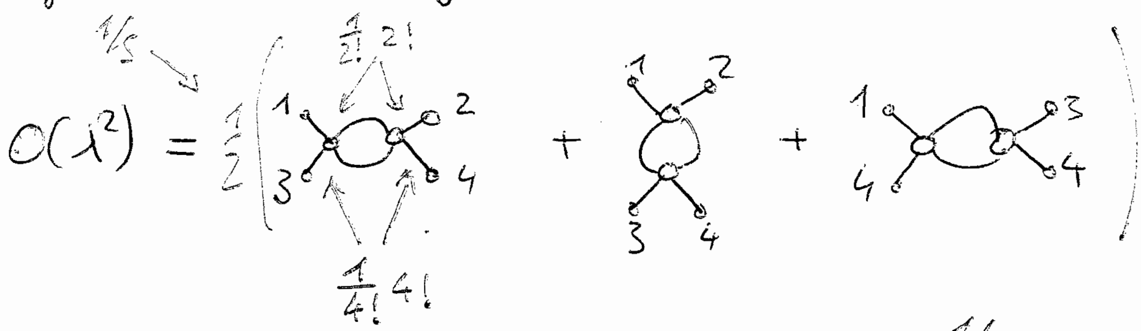


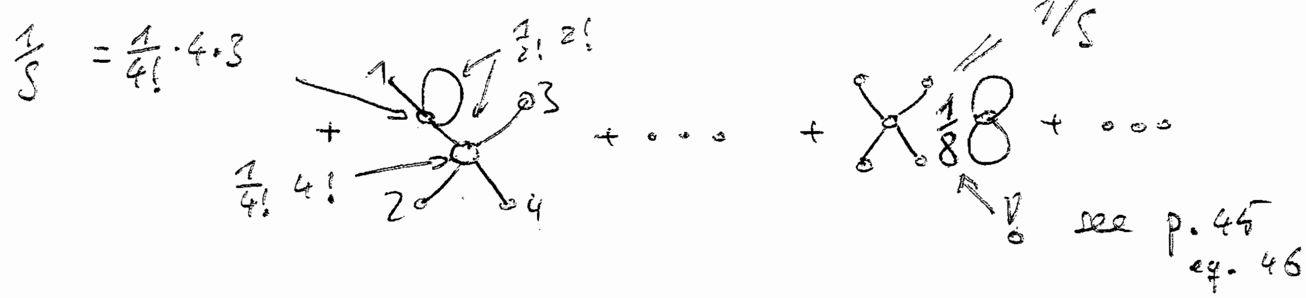
where $\times$ stands for $\left[-i\lambda\int d^4x\right]$

(iii) $O(\lambda^2)$:

$$\left(\frac{-i\lambda}{4!}\right)^2\int d^4x\int d^4z\,\langle 0|\phi_1\cdots\phi_4\;\phi(x)^4\;\phi(z)^4|0\rangle$$

Diagrammatically:





Combinatorics: Taylorexpansion $\frac{1}{n!}H_{int}^n$

(i) $\frac{1}{S}$ $\frac{1}{4!}$ $4!$ $\frac{1}{n!}$ $n!$ ← permut. of $H_{int} \cdots H_{int}$ n-times

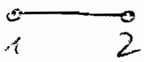
prefactor in H_{int} — permutations of how to contract ϕ^4 in H_{int} with 4 <u>other</u> ϕ's

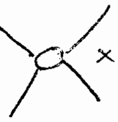
(ii) Symmetry factor S:

Simple comb. of (i) is changed if loops are present: ϕ's in ϕ^4 cannot be counted as "other ϕ's"

$\Rightarrow$ $1/S$ with $S = \#$ of interchanging components without changing diagram

Feynman rules: comp. of $\langle 0|T\phi_1\cdots\phi_n e^{-i\int d^4x\phi^4}|0\rangle$

(i)  $= D_F(x_1-x_2)$

(ii)  $= (-i\lambda)\int d^4x$

(iii) multiplication with $1/S$

Use for final result for ($\langle\Omega|\mathcal{O}|\Omega\rangle =: \langle\mathcal{O}\rangle$)

$$\frac{\langle 0|T\phi_1\cdots\phi_n\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle}{\langle 0|T\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle} =: \langle T\phi_1\cdots\phi_n\rangle$$

For the comput. we note that the each term $\langle 0|T\phi_1\cdots\phi_n\mathcal{L}_{int}{}^n/n!|0\rangle$ can be ordered in terms of contractions between the ϕ_i and the $\mathcal{L}_{int}$'s:

$$\langle 0|T\phi_1\cdots\phi_n\mathcal{L}_{int}{}^n/n!|0\rangle = \langle 0|T\phi_1\cdots\phi_n|0\rangle\langle 0|T\mathcal{L}_{int}{}^n|0\rangle\frac{1}{n!}$$
$$+ \text{"}\,T\phi_1\cdots\overline{\phi_n\mathcal{L}_{int}}|0\rangle\langle 0|T\mathcal{L}_{int}{}^{n-1}|0\rangle\frac{1}{(n-1)!}+\cdots \qquad (3.61)$$

that is,

$$\langle 0|T\phi_1\cdots\phi_n\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle$$

$$= \Big(\langle 0|T\phi_1\cdots\phi_n|0\rangle + \langle 0|T\,\text{``}\phi_1\overline{\cdots\phi_n\,\mathcal{L}_{int}}\text{''}|0\rangle + \langle 0|T\,\text{``}\phi_1\overline{\cdots\phi_n(\mathcal{L}^2_{int}}/2)\text{''}|0\rangle + \cdots\Big)\langle 0|T\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle \tag{3.62}$$

!

It follows that

$$\frac{\langle 0|T\phi_1\cdots\phi_n\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle}{\langle 0|T\, e^{i\int d^4x\,\mathcal{L}_{int}}|0\rangle} = \langle 0|T\,\text{``}\phi_1\overline{\cdots\phi_n\, e^{i\int d^4x\,\mathcal{L}_{int}}}\text{''}|0\rangle \tag{3.63}$$

= all diagrams <u>without</u> vacuum bubbles

Most computations are done in momentum space: Fourier transforms

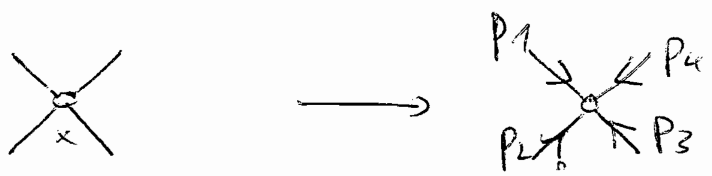
(i) $$D_F(x-y) \longrightarrow \underbrace{\frac{i}{p_1^2 - m^2 + i\varepsilon}}_{\mathcal{D}_F(p)} (2\pi)^4 \delta^4(p_1 - p_2)$$

$p_1 \quad p_2$

(slight abuse of notation)

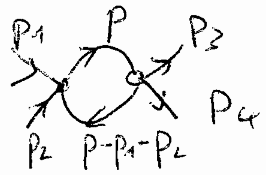
(ii) $$-i\lambda \int d^4x \, \phi(x)^4 = -i\lambda \int \prod_{i=1}^{4} \frac{d^4 p_i}{(2\pi)^4} \, \phi(p_i) \cdot (2\pi)^4 \delta^4(p_1 + p_2 + p_3 + p_4)$$

$$-i\lambda \longrightarrow -i\lambda \, (2\pi)^4 \delta^4(p_1 + p_2 + p_3 + p_4)$$



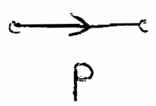
$p_4 = -(p_1 + p_2 + p_3)$

For example:

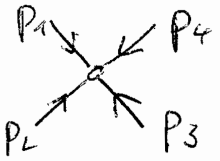


$$= (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \cdot \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\varepsilon} \frac{1}{(p - p_1 - p_2)^2 - m^2 + i\varepsilon}$$

Feynman rules in momentum space

(i) 

$= \dfrac{i}{p^2 - m^2 + i\varepsilon}$

(ii) 

$= -i\lambda$ and $p_4 = -(p_1 + p_2 + p_3)$

momentum conserv.

(iii) $\int \frac{d^4 p}{(2\pi)^4}$ for each loop

(iv) $(2\pi)^4 \, \delta^4 (\sum_i p_i)$ for 

(v) multiplication with $1/S$

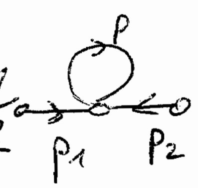
Examples:

1) two-point fct.:

$$\langle T \phi(p_1) \phi(-p_2) \rangle = \frac{i}{p_1^2 - m^2 + i\varepsilon} (2\pi)^4 \delta^4(p_1 + p_2)$$

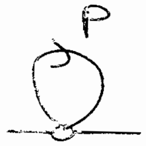
$\parallel$

$(2\pi)^4 \delta^4(p_1 + p_2) \cdot$ 

$+ \frac{1}{2}$ 

$+ \mathcal{O}(\lambda^2)$

$1/S$



without ext. props.

$$= -i\lambda \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon} = -i\Pi + O(\lambda^2) \qquad (3.64)$$

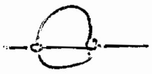
Heuristics:

$$= \frac{i}{p^2 - m^2 + i\varepsilon} + \frac{i}{p^2 - m^2 + i\varepsilon} \, (-i\Pi) \, \frac{i}{p^2 - m^2 + i\varepsilon} + O(\lambda^2)$$

$$= \frac{i}{p^2 - m^2 - \Pi + i\varepsilon} + O(\lambda^2) \qquad (3.65)$$

$\Rightarrow \quad m^2 - \Pi$ = interacting mass

finite!

in general (beyond 1-loop): e.g. 

$$\Pi \to \Pi(p)$$

Remarks: proper treatment: renormalisation

interacting vs free observables: LSZ-form.

Lehman, Symanzik, Zimmermann