8 The Standard model

Electro-weak theory $U(1) \times SU(2)$

& Quantum Chromo dynamics $SU(3)$

8.1 Non-Abelian gauge theories

Consider fermions with $(\partial_\mu U = 0)$

$$\psi(x) \to U\,\psi(x) \quad \text{with } U \in SU(N) \tag{8.1}$$

that is $\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}$, ψ_i Dirac fermion (with four compo.) (8.2)

ψ is in the fundamental representation

Free Dirac action: invariant under (8.1)

$$S_0[\psi] = \int d^4x \, \underbrace{\bar{\psi}(i\partial\!\!\!/ - m)\psi}_{\bar{\psi}_i(i\partial\!\!\!/ - m)\psi_i} \tag{8.3}$$

Global invariance: $\overline{\Psi} \xrightarrow{\mathcal{U}} \overline{\Psi}\,\mathcal{U}^{+}$, $\mathcal{U}$ commutes with γ^0

$$S_D[\mathcal{U}\Psi] = \int d^4x\, \overline{\Psi}\, \mathcal{U}^{+} (i\partial\!\!\!/ - m)\, \mathcal{U}\Psi$$

$$= \int d^4x\, \overline{\Psi} (i\partial\!\!\!/ - m) \underbrace{\mathcal{U}^{+}\mathcal{U}}_{\mathbb{1}_{SU(N)}} \Psi \qquad (8.4)$$

Gauge invariance: invariance of S under

$$\Psi(x) \to \mathcal{U}(x)\, \overline{\Psi}(x) \qquad (8.5)$$

Analogously to QED ($\mathcal{U}(1)$)

we introduce the covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + i A_\mu \qquad (8.6)$$

gauge field (relat. minus sign to eq. (6.9))

$A_\mu \in$ [Lie-algebra $su(N)$]

and

$$S_D[\Psi, A] = \int d^4x\, \overline{\Psi} (i\not{D} - m)\Psi \qquad (8.7)$$

Invariance of $S_D[\psi, A]$ under the gauge transformation (8.5) enforces

$$D_\mu \xrightarrow{\mathcal{U}} \mathcal{U} \mathcal{D}_\mu \mathcal{U}^\dagger \tag{8.8}$$

or (with $\mathcal{U} \mathcal{D}_\mu \mathcal{U}^\dagger = \partial_\mu + i A_\mu^{\mathcal{U}}$)

$$A_\mu(x) \to \mathcal{U}(x) A_\mu(x) \mathcal{U}^\dagger(x) - i\, \mathcal{U}(x) \partial_\mu \mathcal{U}^\dagger(x) \tag{8.9}$$

A_μ has to live in the algebra of $SU(N)$

We write $\mathcal{U} = e^{i\omega}$ (8.10)

Group   algebra

and infinitesimally

$$A_\mu \to A_\mu + i[\omega, A_\mu] - \partial_\mu \omega + O(\omega^2)$$

$$= A_\mu - \mathcal{D}_\mu \omega + O(\omega^2) \tag{8.11}$$

adjoint repr.

Gauge field action: (see p. 135 for $U(1)$)

Fieldstrength:

$$F_{\mu\nu} = \frac{1}{i}[\mathcal{D}_\mu, \mathcal{D}_\nu]$$

$su(N)$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + i\underbrace{[A_\mu, A_\nu]}_{\neq 0} \tag{8.12}$$

Matrix $\downarrow$

$$F_{\mu\nu} = F^a_{\mu\nu}\, t^a \tag{8.13}$$

generators of $SU(N)$

They satisfy $[t^a, t^b] = i f^{abc} t^c$

e.g. $f^{abc} = \varepsilon^{abc}$ in $SU(2)$ structure constants

(8.14)

Gauge transformations: $\mathcal{D}_\mu \to U \mathcal{D}_\mu U^+$

$$F_{\mu\nu} \to U F_{\mu\nu} U^+ \tag{8.15}$$

$\Rightarrow$ Gauge invariant action: $\operatorname{tr} t^a t^b = \frac{1}{2}\delta^{ab}$

$$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x \,\operatorname{tr} F_{\mu\nu}F^{\mu\nu} \tag{8.16}$$

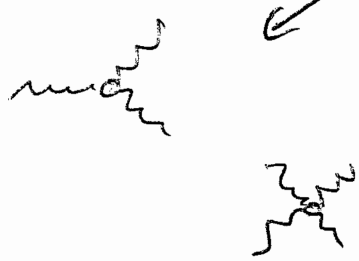
Yang-Mills

In summary we have

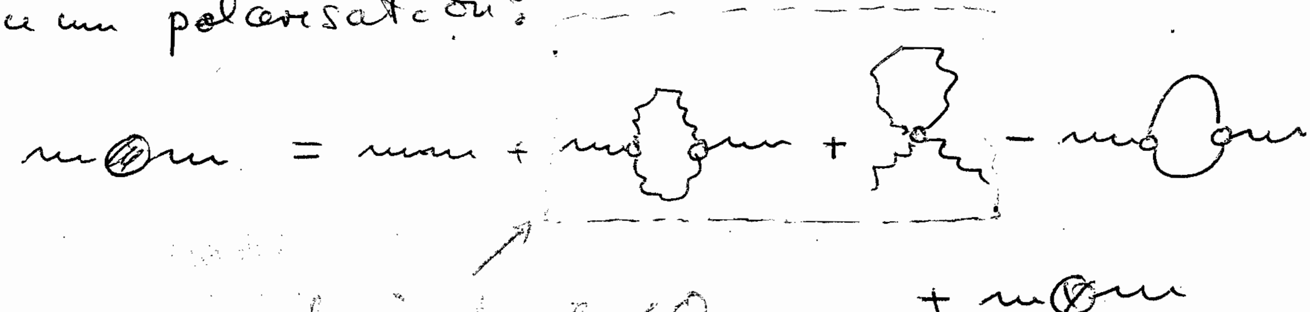
$$S[A,\psi] = S_D[A,\psi] + S_{YM}[A] + \underbrace{S_{gf}[A] + \dots}_{\text{ghosts}} \tag{8.17}$$

Difference to $U(1)$: self interaction of gauge fields!

$$\frac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\Big(\partial_\mu A_\nu^a\, \partial^\mu A^{\nu a} - \partial_\mu A_\nu^a\, \partial^\nu A^{\mu a}\Big)$$
$$- i f^{abc} A_\mu^a A_\nu^b\, \partial^\mu A^{\nu c}$$
$$+ \frac{1}{4} f^{abc} f^{ade} A_\mu^b A_\nu^c A_\mu^d A_\nu^e \tag{8.18}$$



Vacuum polarisation:



dominates ; $\beta < 0$