6 QED

Quantum Electro Dynamics

Particle content:

			Field
Dirac fermions / Leptons	electrons, positrons	: e^-, e^+	ψ_e
	myons	: μ^-, μ^+	ψ_μ
	tau	: τ^-, τ^+	ψ_τ
Gauge bosons	photons	: γ	A_μ

The photon is the gauge boson of the $U(1)$-symmetry with Noether charge: electric charge

see chapter 5.

6.1 Action & Feynman rules

The action is a sum of the Dirac actions of e, μ, τ and the gauge field action of the photon (see eq. (5.16)):

$$S_{QED}[A, \psi_e, \psi_\mu, \psi_\tau]$$

$$= S_D[A, \psi_e] + S_D[A, \psi_\mu] + S_D[A, \psi_\tau]$$

$$+ S_A[A] + S_{gf}[A] \qquad (6.1)$$

with

$$S_D[A, \psi_e] = \int d^4x \, \overline{\psi}_e (i \not{D} - m_e) \psi_e$$

$$D_\mu = \partial_\mu - ie A_\mu \qquad (6.2)$$

and

$$S_A[A] = -\frac{1}{4} \int d^4x \, F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (6.3)$$

The gauge fixing term $S_{gf}[A]$ in covariant gauge is

$$S_{gf}[A] = -\frac{1}{2\xi}\int d^4x \, (\partial_\nu A^\nu)^2 \quad (6.4)$$

with gauge fixing parameter ξ.

Gauge transformations:

$$\psi(x) \to e^{i\alpha(x)}\,\psi(x) = \psi^\alpha(x)$$

$$(6.5)$$

$$A_\nu(x) \to A_\nu(x) + \frac{1}{e}\partial_\nu\alpha(x) = A_\nu^\alpha(x)$$

with

$$S_{QED}[A^\alpha, \psi^\alpha] = S_{QED}[A, \psi]$$

$$+ \frac{1}{\xi}\frac{1}{e}\int d^4x \, \partial_\nu A^\nu \partial_\rho \partial^\rho \alpha$$

$$(6.6)$$

where $\psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$

Feynman rules:

$$S_{QED} = S_{free} + S_I \tag{6.7}$$

with

$$S_{free} = S_A[A] + \int d^4x\, \overline{\Psi}(i\not{\partial} - m)\Psi \tag{6.8}$$

and

$$S_I = e\int d^4x\; \overline{\Psi}\not{A}\,\Psi \tag{6.9}$$

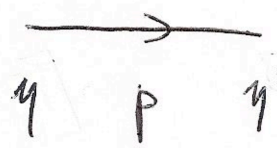
Remark: Any other coupling of leptons and photon introduces dimension-full couplings to the theory, e.g.

$e/\Lambda\ \overline{\Psi}\,\sigma^{\mu\nu}\Psi\ F_{\mu\nu}\Psi$ spin-coupling

where Λ carries momentum dimension one. Such a term makes the theory non-renormalisable & pert. theory fails (at high energies)

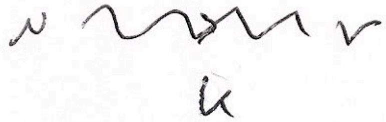
Propagators:

Leptons: (eq. 4.91, p. 127)



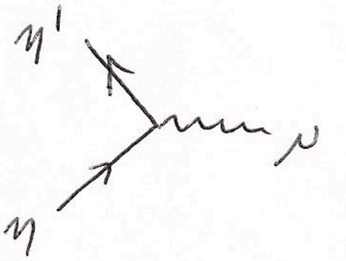
$$= i\left(\frac{\not{p} + m_\varphi}{p^2 - m_\varphi^2 + i\varepsilon}\right)_{\eta\eta'}\,, \quad m_\varphi = m_e, m_\mu, m_\tau \tag{6.10}$$

photon: (eq. 5.56, p. 150)



$$= -\frac{i}{k^2 + i\varepsilon}\left(\eta_{\mu\nu} - (1-\xi)\,\frac{k_\mu k_\nu}{k^2}\right) \tag{6.11}$$

Vertex: (see eq. (4.52), p. 149)



$$= i e \left(\gamma_\mu\right)_{\eta'\eta} \tag{6.12}$$

sign irrelevant ($A_\mu \to -A_\mu$)

The vertex eq. (6.12) has been deduced simply by analogy to the derivation of the scalar self-interaction.

incoming lepton: (eq. (4.93), p. 127)

$$\underset{p}{\circ\!\!\longleftarrow} = u(p) \tag{6.13}$$

out going lepton: (eq. (4.94), p. 128)

$$\longleftarrow\!\!\circ = \bar{u}(p) \tag{6.14}$$

incoming anti-lepton (eq. (4.95), p. 128)

$$\underset{\overleftarrow{p}}{\circ\!\!\longrightarrow} = v(p) \tag{6.15}$$

out going anti-particle (eq. (4.96), p. 128)

$$\underset{\overleftarrow{p}}{\longrightarrow\!\!\circ} = \bar{v}(p) \tag{6.16}$$

Remember: minus-sign for fermion loops (eq. 4.97, p. 128)

incoming photon

$$\underset{k}{\nu\sim\!\circ} = \varepsilon_\nu(k)$$

outgoing photon

$$\circ\!\sim = \varepsilon^*_\nu(k)$$