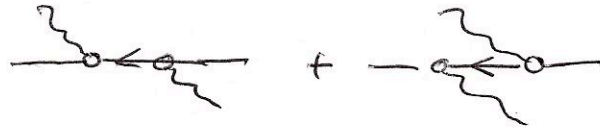


6.2 Elementary processes

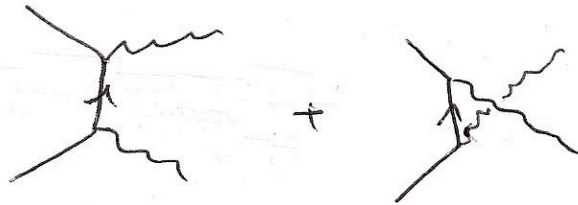
(i) Compton scattering: $e^- \gamma \rightarrow e^- \gamma$



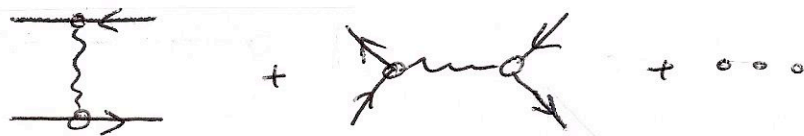
(ii) Elastic $e^- e^-$ -scattering



(iii) pair-annihilation/creation: $e^+ e^- \rightarrow \gamma \gamma$

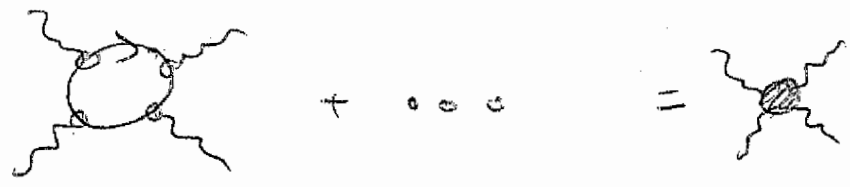



(iv) Bhabha-scattering: $e^+ e^- \rightarrow e^+ e^-$



(i) - (iv) tree level processes

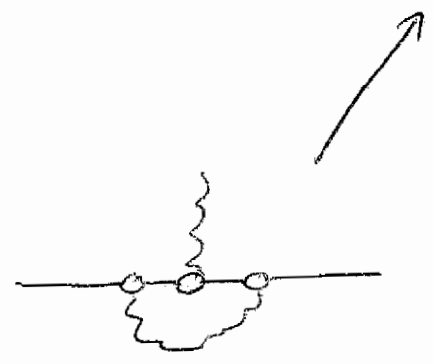
(v) light-by-light scattering
 (non-linear electrodynamics)



 is an effective four photon vertex

(vi) Landé factor (gyromagnetic ratio)

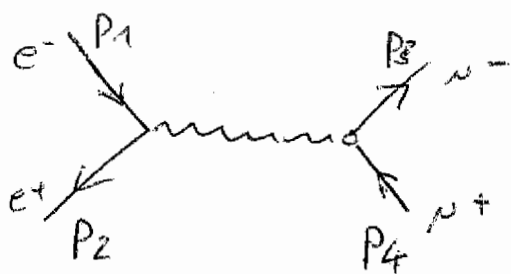
$$i\not{D} - m_e \rightarrow i\not{D} - m_e + \frac{\Delta g}{2} \frac{e}{4m_e} \sigma_{\mu\nu} F^{\mu\nu} \quad (6.17)$$



$$\Delta g = \frac{\alpha}{\pi} \quad (6.18)$$

(v), (vi) loop effects

Tree level example: $e^+e^- \rightarrow \nu^+\nu^-$



only one diagram

2-2 scattering, eq. (3.93) in highly rel. case

$$d\sigma = \int \frac{1}{2s} |M|^2 d\mathcal{V}_2 \quad (6.19)$$

$$\text{with } \int d\mathcal{V}_2 = \frac{1}{2} \frac{1}{(2\pi)^2} \frac{1}{4 p_3^0 p_4^0} d\Omega \frac{s}{4} \quad (6.20)$$

$$\text{where } s = (p_1 + p_2)^2$$

$$|M|^2 = \frac{1}{2} \sum_{\tau} \frac{1}{2} \sum_{\tau'} \sum_{\substack{s, s' \\ \text{sum}}} |M(\tau, \tau', s, s')|^2 \quad (6.21)$$

average
sum

The scattering amplitude is read-off from the

Feynman rules:

$$iM = \bar{u}_{\nu_s}(p_3) (ie\gamma_\mu) v_{\nu_{s'}}(p_4) \left[\frac{-\eta^{\mu\nu}}{s} \right] \bar{v}_{e_{r'}}(p_2) (ie\gamma_\nu) u_{e_r}(p_1) \quad (6.22)$$

It follows that

$$|\mathcal{M}|^2 = \frac{e^4}{4s^2} \cdot T_{\mu\alpha\beta} T_e^{\alpha\beta} \quad (6.23)$$

with

$$T_{\mu\alpha\beta} = \sum_{s,s'} \bar{u}_s(p_3) (ie\gamma_\alpha) v_{\mu s'}(p_4) \cdot \left(\bar{u}_s(p_3) (ie\gamma_\beta) v_{\mu s'}(p_4) \right)^*$$

$$T_e^{\alpha\beta} = \sum_{r,r'} \bar{v}_{e r'}(p_2) (ie\gamma^\alpha) u_{e r}(p_1) \left(\dots \gamma^\beta \dots \right)^* \quad (6.24)$$

We use that (p. 25-33 in e^+e^- to $\mu^+\mu^-$)

$$\sum_s u_{s'}(p_3) \bar{u}_s(p_3) = \not{p}_3 + m_\mu \leftarrow \text{eq. (4.67)}$$

$$\sum_{s,s'} \bar{u}_s(p_3) \gamma_\alpha \left[v_{\mu s'}(p_4) \bar{v}_{\mu s'}(p_4) \right] \gamma_\beta^* u_s(p_3)$$

$$\sum_{s'} v_{\mu s'}(p_4) \bar{v}_{\mu s'}(p_4) = \not{p}_4 - m_\mu \leftarrow \text{eq. (4.67)}$$

$$= \text{Tr} \left(\not{p}_3 + m_\mu \right) \gamma_\alpha \left(\not{p}_4 - m_\mu \right) \gamma_\beta \quad (6.25)$$

with

$$\begin{aligned} \left[\bar{u}_s(p) \gamma_\alpha v_{s'}(q) \right]^* &= \underbrace{v_{s'}^\dagger(q)}_{\bar{v}_s(q)} \underbrace{\gamma^0 \gamma_\alpha \gamma^0}_{\gamma_\alpha} \underbrace{\gamma^0}_{\gamma^0} \underbrace{u_s^\dagger(p)}_{u_s(p)} \\ &= \bar{v}_s(q) \gamma_\alpha u_s(p) \end{aligned} \quad (6.26)$$

Highly relativistic limit: drop m_ν, m_e

$$\Rightarrow T_{\nu\alpha\beta} = \text{Tr}(\not{p}_3 + m_\nu) \gamma_\alpha (\not{p}_4 - m_\nu) \gamma_\beta$$

$$\text{Tr} \gamma^{2n+1} = 0 \rightarrow = \text{Tr} \not{p}_3 \gamma_\alpha \not{p}_4 \gamma_\beta + \text{Tr} \gamma_\alpha \gamma_\beta m_\nu^2 \quad (6.27)$$

We use

$$(a) \quad \text{Tr} \gamma^\rho \gamma^\sigma = \frac{1}{2} \text{Tr} \{ \underbrace{\gamma^\rho, \gamma^\sigma}_{2\eta^{\rho\sigma}} \} = 4\eta^{\rho\sigma}$$

$$(b) \quad \text{Tr} \gamma^\rho \gamma^\sigma \gamma^\alpha \gamma^\beta = \underbrace{2\eta^{\rho\sigma} \text{Tr} \gamma^\alpha \gamma^\beta}_{8\eta^{\rho\sigma} \eta^{\alpha\beta}} - \text{Tr} \gamma^\sigma \gamma^\rho \gamma^\alpha \gamma^\beta$$

$$= \dots$$

$$\Rightarrow \text{Tr} \not{p}_3 \gamma^\sigma \not{p}_4 \gamma^\alpha \gamma^\beta = 4 (\eta^{\rho\sigma} \eta^{\alpha\beta} - \eta^{\rho\alpha} \eta^{\beta\sigma} + \eta^{\rho\beta} \eta^{\alpha\sigma}) \quad (6.28)$$

and hence:

$$T_{\nu\alpha\beta} = 4 (p_{3\alpha} p_{4\beta} + p_{3\beta} p_{4\alpha} - \eta_{\alpha\beta} p_3 \cdot p_4) - 4 \eta_{\alpha\beta} m_\nu^2$$

$$\text{as } m_\nu^2 \rightarrow 0 \quad (6.29)$$

We similarly compute

$$T_e^{\alpha\beta} \approx 4 (p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha) \quad (6.30)$$

and arrive at

$$\begin{aligned} |M|^2 &= \frac{e^4}{4s^2} 2 \cdot 16 \left[(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4) \right] \\ &= \frac{8e^4}{s^2} \left[\quad \right] \end{aligned} \quad (6.31)$$

and in summary, after inserting (6.31) in (6.19)

$$\frac{d\sigma}{d\Omega} = \frac{2\alpha^2}{p_3^0 p_4^0 s} \left[(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4) \right] \quad (6.32)$$

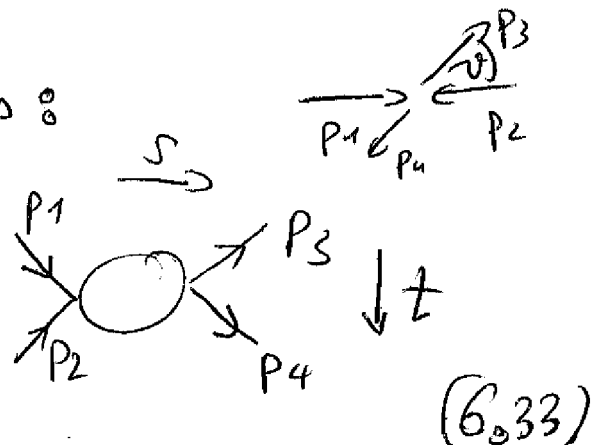
with $\alpha = e^2/4\pi$

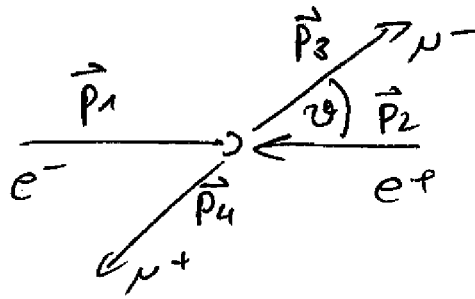
Mandelstam variables:

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$





scattering angle θ :

$$\cos \theta = \frac{\vec{p}_1 \cdot \vec{p}_3}{|\vec{p}_1| |\vec{p}_3|}$$

highly relativistic limit

$$p_1 \cdot p_3 = p_1^0 p_3^0 - \vec{p}_1 \cdot \vec{p}_3 \approx \frac{1}{4} s (1 - \cos \theta)$$

$$\approx \frac{1}{4} s \quad \approx \frac{1}{4} s \cos \theta$$

$$= p_2 \cdot p_4$$

$$p_1 \cdot p_4 = p_2 \cdot p_3 = \frac{1}{4} s (1 + \cos \theta)$$

$$\Rightarrow (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)$$

$$= \frac{1}{16} s (2 + 2 \cos^2 \theta) = \frac{1}{8} s (1 + \cos^2 \theta)$$

The final result for $|M|^2$ is

$$|M|^2 = e^4 (1 + \cos^2 \theta) = 16\pi^2 \alpha^2 (1 + \cos^2 \theta)$$

with $\alpha = e^2/4\pi$. (6.34)

Eq. (6.34) has to be compared with that for the scalar 2-2 scattering, eq. (3.23), p. 45, $|M|^2 = A^2$.

We insert eq. (6.34) in eq. (6.19) for the cross section, ($4p_3^0 p_4^0 \simeq s$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \quad (6.35)$$

Remarks:

(i) The QED $e^+e^- \rightarrow \mu^+\mu^-$ cross section

$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2\theta)$ should be compared with scalar cross section $\phi\phi \rightarrow \phi\phi$,

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2} \frac{\lambda^2}{4s}, \text{ eq. (3.93), p. 80.}$$

(ii) The intermediate virtual photon was

chosen in Feynman gauge, $\xi=1$. We

have shown on p. 151, eq. (5.58) that

any choice of ξ leads to the same

result, in particular $\xi=0$: $\sum_{\nu} \frac{v_{\nu} v_{\nu}}{k^{\nu}} = 0$.

(iii) In the high energy limit also

$$(p_1 - p_2)_{\nu} \bar{v}(p_2) \gamma^{\nu} u(p_1) \stackrel{\sim m_e}{\approx} 0, \text{ only}$$

the physical polarisations ϵ_1, ϵ_2 play

a role, ϵ_3 drops out. See eq. (5.34), (5.35).

(iv) The argument done in (iii) also applies to $\bar{u}(p_3) \gamma^\nu v(p_4)$. In summary

$$\text{we have } \bar{u}(p_3) \gamma^\nu v(p_4) p_{3\mu} v \approx 0$$

$$(\bar{v}(p_2) \gamma^\nu u(p_1) p_{1\mu} v \approx 0)$$

So if $p_{3,4}$ are orthogonal to the beam axis, defined by $p_{1,2}$, the related polarisation

ϵ_1 or ϵ_2 also 'drops out of the game'.

In this case, $\alpha = \pi/2$, only one polarisation contributes to the scattering, for $\alpha = 0$, both.

(v) In the highly-relativistic case and

$$\alpha = \pi/2 \quad \text{u} \quad \text{v} \rightarrow \text{u} \quad \text{v}$$