Problems for Quantum Field Theory 1

5. Sheet

Suggested reading before solving these problems: Chapters 3.1 to 3.2 in the script and/or Chapters 4.2 to 4.3 of *Peskin & Schroeder*.

Problem 1: *Time and normal ordering 1*

Explain in a few words and formula

a) What is a time ordered product and why do we need it?

b) What is a normal ordered product and why is it useful?

c) How can we transform a time ordered expression to a normal ordered one?

Problem 2: *Time and normal ordering 2*

a) Show that the time ordered product $T(\phi(x_1)\phi(x_2))$ and the normal ordered product $:\phi(x_1)\phi(x_2):$ are both symmetric under the interchange of x_1 and x_2. Deduce that the Feynman propagator $D_F(x_1 - x_2)$ has the same property.

b) Check Wick's theorem for the case of three real scalar fields

$$\begin{aligned} T(\phi(x_1)\phi(x_2)\phi(x_3)) &= :\phi(x_1)\phi(x_2)\phi(x_3): + \phi(x_1)D_F(x_2 - x_3) \\ &\quad + \phi(x_2)D_F(x_1 - x_3) + \phi(x_3)D_F(x_1 - x_2). \end{aligned}$$

Problem 3: *Three-point correlation function*

Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - h\Phi\phi\phi.$$

The last term is an interaction term that allows a Φ particle to decay into two ϕ's, provided that $M > 2m$. Assume that this is the case and calculate the $\mathcal{O}(h)$ contribution to the correlation function

$$\langle 0| T\ \Phi(x)\,\phi(y)\,\phi(z)\ e^{-i\int dt\ H_{\text{int}}(t)}|0\rangle.$$