Problems for Quantum Field Theory 1

6. Sheet

Suggested reading before solving these problems: Chapters 3.3 to 3.4 in the script and/or Chapters 4.4 to 4.6 of *Peskin & Schroeder*.

Problem 1: *Key figures of LHC*

As you might have heard, the LHC has started operation on November 20. The protons circulating in the accelerator (the proton beam) are divided into 2808 bunches (and similar for the antiprotons). Find out

a) How many particles will be in one bunch?

b) What is the maximal energy per proton and what is therefore the energy of the whole proton beam? How fast could one accelerate a ICE (400 000 kg) with that energy?

c) What is the width of a bunch at the collision point?

d) The Luminosity is the event rate for a process divided by the corresponding cross section $L = \dot{N}/\sigma$. What follows from the numbers obtained in parts a) and d) for L? (The accelerator ring is 27 km long, the velocity of a particle is $0.9999c$.)

Problem 2: *Scattering matrix and cross-section*

On sheet 5 you considered the following theory involving two real scalar fields:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 - h\Phi\phi\phi.$$

a) Write down the Feynman rules in momentum space for this theory.

b) Consider a scattering experiment where two ϕ-particles with initial momenta $\vec{k}_1$ and $\vec{k}_2$ are scattered to final momenta $\vec{p}_1$ and $\vec{p}_2$. Show that at order h^2 there are 3 tree-level[1] contributions to the scattering matrix:

$$i\mathcal{M} = \frac{-i(2h)^2}{(k_1+k_2)^2 - M^2} + \frac{-i(2h)^2}{(k_1-p_1)^2 - M^2} + \frac{-i(2h)^2}{(k_1-p_2)^2 - M^2}.$$

c) Derive an expression for the differential cross section $d\sigma/d\Omega$ in the center-of-mass frame as a function of the angle ϑ between $\vec{k}_1$ and $\vec{p}_1$.

[1] A tree-level process has a Feynman diagram without loops.

d) Consider the limit $h^2 \to \infty$, $M^2 \to \infty$ with fixed ratio

$$\frac{1}{4!}\lambda = -\frac{1}{2}\frac{h^2}{M^2}.$$

Calculate the total cross section σ in this limit. Do not forget to take into account that the final state has two identical particles.

d) Show that the result for σ is identical to the cross-section of ϕ^4 theory calculated at tree-level and to first order in the interaction parameter λ.