

PROBLEMS FOR QUANTUM FIELD THEORY 1  
3. Tutorial

PROBLEM 1: *Lorentz invariance of the 3-momentum integral*

In the lecture we have used the following, very important identity for the three-momentum integral

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_{\vec{p}}} f(p) = \int \frac{d^4p}{(2\pi)^4} 2\pi \delta(p^2 - m^2) \Theta(p_0) f(p),$$

with  $\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$ . Note that

$$\delta(g(x) - g(a)) = \frac{1}{|g'(a)|} \delta(x - a).$$

Convince yourself that this implies that the three-momentum integral is invariant under proper, orthochronous Lorentz transformations  $\Lambda \in SO(1, 3)$  if  $f(\Lambda p) = f(p)$ . Which functions  $f$  satisfy  $f(\Lambda p) = f(p)$ ?