Problems for Quantum Field Theory 1
4. Tutorial

PROBLEM 1: *Connection between operators in different pictures*

Consider the decomposition of the Hamiltonian operator H in terms of free and interaction parts, $H = H_0 + H_{\text{int}}$. You have learned that, in the Heisenberg picture, operators evolve in time with the *full* Hamiltonian H, while in the interaction picture they evolve with the *free* Hamiltonian H_0.

In the lecture we have derived a relation between time-ordered expectation values of products of fields in the Heiselberg picture and the interaction picture. To that end we have used the relation between the scalar field in the Heisenberg picture ϕ_H, and in the interaction picture, ϕ_I. Show that they are connected by

$$\phi_H(t,\, \vec{x}) \;=\; U^\dagger(t,0)\, \phi_I(t,\vec{x})\, U(t,0)$$

where the time evolution operator $U(t,0)$ is an unitary operator that you should express in terms of H_0 and H,

$$U(t,0) = e^{iH_0 t}\, e^{-iHt}\,. \tag{1}$$

Show (1) by showing that $U(t,0)$ satisfies the defining equation, $i\partial_t U(t,0) = H_I(t)U(t,0)$. Here $H_I(t) = H_{\text{int}}(t) = e^{iH_0 t} H_{\text{int}} e^{-iH_0 t}$.