Problems for Quantum Field Theory 1
7. Tutorial

Problem 1: *Lie groups and Lie algebras*
The finite elements of a connected Lie group can be obtained from an infinite product
of transformations infinitesimal close to $\mathbb{1}$. A well known example is the group $SO(3)$
of rotations in $\mathbb{R}^3$. Show that for a matrix M

$$\lim_{N \to \infty} \left(\mathbb{1} + \frac{\alpha}{N} M \right)^N = \exp(\alpha M) \tag{1}$$

by comparing the Taylor series in α. The expression in Eq. (1) is an element of a
matrix Lie group while the matrix M is part of the corresponding Lie algebra.