Problems for Quantum Field Theory 1
9. Tutorial

Problem : *Representations of Clifford algebra*

Examine the following representation of the Clifford algebra

$$\Gamma^0 \;=\; \gamma^0\gamma^2 \qquad \Gamma^1 = i\gamma^0\gamma^1$$
$$\Gamma^2 \;=\; i\gamma^0 \qquad \Gamma^3 = i\gamma^0\gamma^3$$

where the γ^μ are the Dirac matrices in the chiral representation. Prove that the Γ^μ) are anti-hermitian, and that they are a representation of the Ciffold algebra,

$$\{\Gamma^\mu,\, \Gamma^\nu\} \;=\; 2\,\eta^{\mu\nu}\,.$$

Express the matrix $\Gamma^5 \;=\; i\,\Gamma^0\Gamma^1\Gamma^2\Gamma^3$ in terms of Dirac matrices γ^μ. Is this matrix hermitian or anti-hermitian? Show that it commutes with the Γ^μ!

Has the Dirac equation, when expressed in this representation, some special property?