# Faddeev - Popov quantisation

| gauge theory | 0-dim |
|---|---|
| $\int dA \; e^{-S[A]}$ | $\int d^2x \; e^{-S(r)}$ |
| $\mathcal{U} = e^{g\omega} \downarrow A^g = \mathcal{U} A_\mu \mathcal{U}^\dagger + \frac{1}{g} \mathcal{U} \partial_\mu \mathcal{U}^\dagger$ | $R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \downarrow x_1^\theta = x_1 \cos\theta + x_2 \sin\theta = \mathcal{F}$ |
| $1 = \int dg \; \delta[\mathcal{F}[A^g]] \cdot \Delta_{\mathcal{F}}[A]$ | $1 = \int_0^{2\pi} d\theta \; \delta(x_1^\theta) \cdot \Delta_{\mathcal{F}}(\vec{x})$ |
| with | with |
| $\Delta_{\mathcal{F}}[A] = \left( \int dg \; \delta[\mathcal{F}[A]] \right)^{-1}$ | $\Delta_{\mathcal{F}}(\vec{x}) = \left( \int_0^{2\pi} d\theta \; \delta(x_1^\theta) \right)^{-1}$ |
| $= \# \sum_{\omega_0} \left| \det \frac{\delta \mathcal{F}}{\delta \omega} \right|_{A = A^{\mathcal{U}(\omega_0)}}$ | $= \left( \int_0^{2\pi} d\theta \frac{1}{\lvert -x_1 \sin\theta + x_2 \cos\theta \rvert} \cdot \left( \delta\left(\theta - \arctan \frac{x_2}{x_1}\right) + \delta\left(\theta - \arctan \frac{x_2}{x_1} + \pi\right) \right) \right)^{-1}$ |
| | $= r/2 \, (\vec{x})$ |
| $\Rightarrow \int dA \; e^{-S[A]}$ | $\Rightarrow \int d^2x \; e^{-S(r)}$ |
| $= \int dA \, dg \; \delta[\mathcal{F}[A^g]] \Delta_{\mathcal{F}}[A] \, e^{-S[A]}$ | $= \int d^2x \int_0^{2\pi} d\theta \; \delta(x_1^\theta) \, \frac{r}{2} \; e^{-S(r)}$ |
| $= \int dA \; \delta[\mathcal{F}[A]] \; \Delta_{\mathcal{F}}[A] \; e^{-S[A]} \cdot \left[\int dg\right]$ | $= \int d^2x \, \delta(x_1) \; r/2 \; e^{-S(r)} \cdot [2\pi]$ |
| $= \int dA_{gf} \; \Delta_{\mathcal{F}}[A_{gf}] \, e^{-S[A_{gf}]} \cdot \left[\int dg\right]$ | $= \int_0^\infty dx_2 \; x_2 \; e^{-S(\sqrt{x_2^2})} \cdot [2\pi]$ |
| | with $x_2 = r\left(\vec{x} = \begin{pmatrix} 0 \\ x_2 \end{pmatrix}\right)$ |

Gribov copies:

$$X_2^{\Theta} = x_2 \cos\Theta - x_1 \sin\Theta \Big|_{\substack{\Theta_+ = \arctan x_2/x_1 \\ \Theta_- = \arctan x_2/x_1 + \pi}} : \quad X_2^{\Theta_+} = -X_2^{\Theta_-}$$

and $r = \pm x_2^{\Theta}$ ($x_2^{\Theta_+}$ can be posit./negative

Remove absolute value:

$$|X_2^{\Theta}| \to x_2^{\Theta}$$

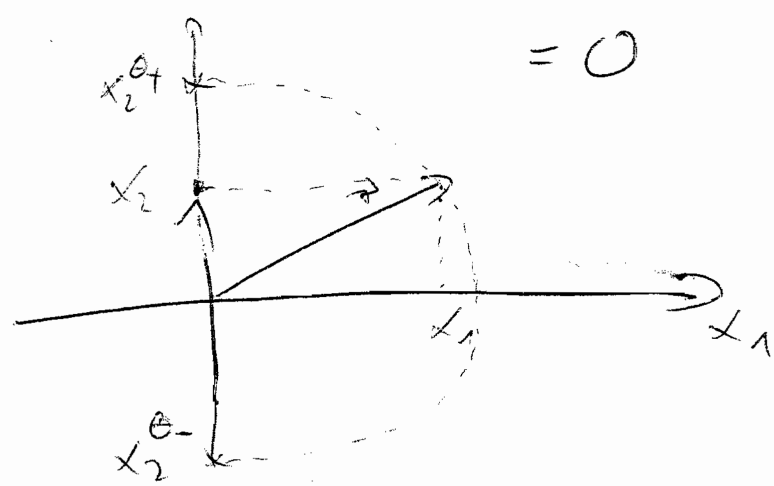
a.) $$\int d\Theta \, \frac{1}{x_2^{\Theta}} \left( \delta(\Theta - \arctan x_2/x_1) + \delta(\Theta - \arctan x_2/x_1 + \pi) \right)$$

$$= 0$$

b.) $$\int d^2x \, x_2^{\Theta}/2 \; \delta(x_1^{\Theta}) \, e^{-S(r)}$$

$$= \int d^2x \, x_2/2 \; \delta(x_1) \, e^{-S(r)} = \int dx_2 \, x_2/2 \, e^{-S(\sqrt{x_2^2})}$$

$$= 0$$



ghosts: assume exactly $\underline{1}$ copy

$\boxed{\frac{\delta \mathcal{F}}{\delta \omega} > 0}$

$$\det \frac{\delta \mathcal{F}}{\delta \omega} = \int d\bar{c}\, dc\, e^{-\int d^d x\, \bar{c} \frac{\delta \mathcal{F}}{\delta \omega} c}$$

II-37a

$$= \int d\bar{c}\, dc\, e^{-S_{gh}[A, c, \bar{c}]}$$

with

$$S_{gh}[A, c, \bar{c}] = \int d^d x\, \bar{c} \frac{\delta \mathcal{F}}{\delta \omega} c$$

cov. gauge $$= - \int d^d x\, \bar{c}^a \partial_\nu D_\nu^{ab} c^b$$

Generating functional: $\mathcal{F}^a \to \mathcal{F}^a - \mathcal{C}^a$

$\Delta_{\mathcal{F}} \to \Delta_{\mathcal{F}}$

$$Z[J, \eta, \bar{\eta}] = \frac{1}{N} \int dA \int d\mathcal{C}\, e^{-\frac{1}{2\xi} \int d^d x\, \mathcal{C}^a(x)^2}$$

$$\int d\bar{c}\, dc\, e^{-S_{gh}[A, c, \bar{c}]}$$

$$\cdot \int d\mathcal{U}\, \delta[\mathcal{F} - \mathcal{C}]\, e^{-S_{YM}[A]}$$

$$\cdot\, e^{\int d^d x \{ J_\nu^a A_\nu^a + \bar{\eta}^a c^a - \bar{c}^a \eta^a \}}$$

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Remark II-37b Neuberger Problem

Fermions - reminder:

Grassmann variables: (1) $c^2 = \bar{c}^2 = 0$

(2) $c\bar{c} = \bar{c}c$

$$\int dc\, c^n = \delta_{n1}$$

$$\Rightarrow \int dc\, f(c) = \left.\frac{\partial f}{\partial c}\right|_{c=0}$$

$$\Rightarrow \int \prod_n d\bar{c}_n\, dc_n\, e^{-\vec{\bar{c}}\, \mathcal{M}\, \vec{c}}$$

$$= \int \prod_{n=1}^{N} d\bar{c}_n\, dc_n\, e^{-\bar{c}_i\, \mathcal{M}_{ij}\, c_j}$$

$$= \int \prod_{n=1}^{N} d\bar{c}_n\, dc_n\, \left(-\bar{c}_i\, \mathcal{M}_{ij}\, c_j\right)^N \frac{1}{N!}$$

$$= \sum_\sigma (-1)^\sigma \prod_i \mathcal{M}_{i\sigma(i)} \qquad = \det \mathcal{M}$$

Neuberger problem:

(BRST)

$$\int dA_{gf} \det \frac{\delta \mathcal{F}}{\delta \omega} \; e^{-S[A_{gf}]} \hat{=} 0$$

Resolutions:

(1) take absolute value:

difficult as gauge fixing is done as extremisation

(2) gauge fixing as top. field theory (Morse theory ...)

(3) do not gauge-fix