

7.3 Callan-Symanzik equation

let us first come back to our generating functional Z with

$$Z = \int [D\phi]_{\Lambda} e^{-S_{\text{eff},\Lambda}[\phi]} \quad (7.46)$$

Note that Z is Λ -independent

$$\Lambda \frac{d}{d\Lambda} Z = 0 \quad (7.47)$$

as it is simply $\int D\phi e^{-S[\phi]}$. Using the

β -functions as defined in eq. (7.30),

eq. (7.45) can be rewritten as ~~no hat, dim. full~~

$$\left(\Lambda \frac{\partial}{\partial \Lambda} + \beta_i \frac{\partial}{\partial \lambda_i} \right) Z = 0 \quad (7.48)$$

which entails the RG-invariance / regularisation

independence of the theory, with

$$\beta_i = \Lambda \frac{d\lambda_i}{d\Lambda} \quad (7.49)$$

Even though we have performed our derivation for Z , it also holds true

$$\text{for } \langle \phi(x_1) \dots \phi(x_n) \rangle = G^{(n)}(x_1, \dots, x_n) \quad (7.50)$$

renormalised Green fct.

However, the reparameterisation step $\phi \rightarrow \frac{1}{Z^{1/2}} \phi$ causes a global Λ -dependence of eq. (7.50)

and hence

$$\left(\Lambda \frac{d}{d\Lambda} + \underbrace{n/2 \frac{1}{Z} \frac{dZ}{d\Lambda}}_{\gamma} \right) G^{(n)} = 0 \quad (7.51)$$

The total Λ -derivative can be rewritten in terms of the β -fcts.:

$$\left(\Lambda \frac{\partial}{\partial \Lambda} + \beta_i \frac{\partial}{\partial \lambda_i} + n/2 \gamma \right) G^{(n)} = 0 \quad (7.52)$$

We remark that we also could have started with the unrenormalised correlation

$$\text{function } G_0^{(n)} = \langle \phi_0 \dots \phi_0 \rangle \quad (7.53)$$

and, by varying the RG-scale μ , at which the RG-conditions are fixed (see QFT I), together with using that $\phi_{ren} = \phi_0 \frac{1}{Z^{1/2}}$, we would have arrived at eq. (7.52) with $\Lambda \rightarrow \mu$. Thus we can map the task of computing β_i to that of computing $\Delta Z, \Delta m^2, \dots$, eq., in dim. reg.

However, beyond perturbation theory, Wilson's approach is far more powerful also for practical purposes.