

PROBLEMS FOR QUANTUM FIELD THEORY 2
5. Sheet

Suggested reading before solving these problems: Chapter 3.1-3.2 in the script and/or chapter 7.5 in *Peskin & Schroeder*.

PROBLEM 1: *The exact two-point function*

- a) Show diagrammatically that one can write the renormalized two point function of some theory as

$$G(p) = G_0(p) + G_0(p)\Pi(p)G_0(p) + G_0(p)\Pi(p)G_0(p)\Pi(p)G_0(p) + \dots, \quad (1)$$

where $\Pi(p)$ is the sum of all one-particle irreducible insertions into the propagator.

- b) Show that the above series can be resummed

$$G(p) = (G_0^{-1}(p) - \Pi(p))^{-1}.$$

- c) Consider now a scalar field φ . What is the relation of $\Pi(p)$ to the renormalized mass $m_{\varphi,r}^2$ and the wavefunction renormalization constant Z_φ defined by the expansion

$$G^{-1}(p) = Z_\varphi(p^2 + m_{\varphi,r}^2) + \mathcal{O}(p^4)$$

for small values of p^2 ?

PROBLEM 2: *Two point function of Yukawa theory*

On the previous exercise sheet you have derived a perturbative expression for the boson two-point function of Yukawa theory

$$G(p) = G_0(p) + G_0(p)\Pi(p)G_0(p) + \dots,$$

with

$$\Pi(p) = -h^2 \int \frac{d^d q}{(2\pi)^d} \frac{-d q \cdot (q + p) + d m^2}{(q^2 + m^2)((q + p)^2 + m^2)},$$

where m is the mass of the fermions. Simplify the expression for $\Pi(p)$ further by introducing Feynman parameters (see Sheet 9 of QFT 1) and derive expressions for the renormalized mass $m_{\varphi,r}^2$ and the coefficient Z_φ for small $\epsilon = 4 - d$.