Problems for Quantum Field Theory 2

8. Sheet

Suggested reading before solving these problems: Chapter 4.3, 4.4 in the script and/or chapter 15.7, 17.1 in *Weinberg: The Quantum Theory of Fields 2*.

Problem 1: *BRST Symmetry and the effective action*
Consider the generating functional for Yang-Mills theory

$$Z[J,\eta,\bar\eta] = \int DADcD\bar{c}Db\, e^{-S[A,c,\bar{c},b]+\int_x\{J\cdot A+\bar\eta\cdot c-\bar{c}\cdot\eta\}}.$$

a) Use the symmetry of the meausure $DADcD\bar{c}Db$ and the action S under BRST transformations to derive

$$\int_x \{J\cdot\langle\epsilon QA\rangle + \bar\eta\langle\epsilon Qc\rangle - \langle\epsilon Q\bar{c}\rangle\eta\} = 0. \tag{1}$$

Why is ist not possible to replace $\langle\epsilon QA\rangle$ by $\epsilon Q\langle A\rangle$ etc. in this expression?

b) Introduce now source terms for QA, Qc and $Q\bar{c}$

$$Z[J,\eta,\bar\eta,L_A,L_c,L_{\bar{c}}] = \int DADcD\bar{c}Db\, e^{-S[A,c,\bar{c},b]+\int_x\{J\cdot A+\bar\eta\cdot c-\bar{c}\cdot\eta+L_A\cdot QA+L_cQc+L_{\bar{c}}Q\bar{c}\}}.$$

Why does Eq. (1) still hold for arbitrary soureses L_A, L_c, $L_{\bar{c}}$? Show that it can now be represented by

$$\int_x\left\{J\cdot\frac{\delta Z}{\delta L_A} - \bar\eta\cdot\frac{\delta Z}{\delta L_c} - \frac{\delta Z}{\delta L_{\bar{c}}}\cdot\eta\right\} = 0. \tag{2}$$

c) Define now the effective action as the Legendre transform of $\ln Z$ according to

$$\Gamma[A,c,\bar{c};L_A,L_c,L_{\bar{c}}] = \int_x\{J\cdot A+\bar\eta\cdot c-\bar{c}\cdot\eta\} - \ln Z[J,\eta,\bar\eta,L_A,L_c,L_{\bar{c}}],$$

with $J=\frac{\delta\Gamma}{\delta A}$, $\bar\eta = -\frac{\delta\Gamma}{\delta c}$, $\eta = -\frac{\delta\Gamma}{\delta\bar{c}}$. Prove the relations

$$\frac{1}{Z}\frac{\delta Z}{\delta L_A} = -\frac{\delta\Gamma}{\delta L_A},\quad \frac{1}{Z}\frac{\delta Z}{\delta L_c} = -\frac{\delta\Gamma}{\delta L_c},\quad \frac{1}{Z}\frac{\delta Z}{\delta L_{\bar{c}}} = -\frac{\delta\Gamma}{\delta L_{\bar{c}}}.$$

d) Finally, show that Eq. (2) leads to the master equation for the effective action

$$\int_x\left\{\frac{\delta\Gamma}{\delta L_A}\cdot\frac{\delta\Gamma}{\delta A} + \frac{\delta\Gamma}{\delta L_c}\cdot\frac{\delta\Gamma}{\delta c} + \frac{\delta\Gamma}{\delta L_{\bar{c}}}\cdot\frac{\delta\Gamma}{\delta\bar{c}}\right\} = 0.$$