

PROBLEMS FOR QUANTUM FIELD THEORY 2
9. Sheet

Suggested reading before solving these problems: Chapter 5.1-5.2 in the script and/or chapter 16.5 in *Peskin & Schroeder*.

PROBLEM 1: *Gauge-Boson self energy*

Consider the self energy of the gauge bosons in pure Yang-Mills theory (i.e. only gluons and ghosts). Due to BRST invariance it is of the form

$$(\Pi^{\mu\nu}(p^2))^{ab} = (p^2 \delta^{\mu\nu} - p^\mu p^\nu) \Pi(p^2)^{ab}.$$

Show that there are three diagrams contributing at one-loop level. Show that the corresponding expressions are in Feynman gauge ($\xi = 1$)

$$(\Pi_1^{\mu\nu}(p^2))^{ab} = \frac{g^2}{2} \int \frac{d^4 l}{(\pi)^4} \frac{1}{l^2(l+p)^2} f^{acd} f^{bcd} N^{\mu\nu}$$

with

$$N^{\mu\nu} = [\delta^{\mu\rho}(p-l)^\sigma + \delta^{\rho\sigma}(2l+p)^\mu + \delta^{\sigma\mu}(-l-2p)^\rho] \\ [\delta^{\nu\rho}(l-p)^\sigma + \delta^{\rho\sigma}(-2l-p)^\nu + \delta^{\nu\sigma}(l+2p)^\rho],$$

and

$$(\Pi_2^{\mu\nu}(p^2))^{ab} = \frac{g^2}{2} \int \frac{d^4 l}{(2\pi)^4} \frac{\delta^{\rho\sigma}}{l^2} \delta^{cd} (M^{\rho\sigma})^{ab\ cd},$$

with

$$(M^{\rho\sigma})^{ab\ cd} = f^{abe} f^{cde} (\delta^{\mu\rho} \delta^{\nu\sigma} - \delta^{\mu\sigma} \delta^{\nu\rho}) \\ + f^{ace} f^{bde} (\delta^{\mu\nu} \delta^{\rho\sigma} - \delta^{\mu\sigma} \delta^{\nu\rho}) \\ + f^{ade} f^{bce} (\delta^{\mu\nu} \delta^{\rho\sigma} - \delta^{\mu\rho} \delta^{\nu\sigma}),$$

as well as

$$(\Pi_3^{\mu\nu}(l^2))^{ab} = (-1)g^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2(l+p)^2} f^{dac} (l+p)^\mu f^{cbd} l^\nu.$$

The expressions for $\Pi_1^{\mu\nu}$, $\Pi_2^{\mu\nu}$ and $\Pi_3^{\mu\nu}$ can be simplified further by using dimensional regularization, introducing Feynman parameters and by using the identity

$$f^{acd} f^{bcd} = C_2(G) \delta^{ab}.$$

Show that the sum $\Pi^{\mu\nu} = \Pi_1^{\mu\nu} + \Pi_2^{\mu\nu} + \Pi_3^{\mu\nu}$ is finite for $d \rightarrow 2$ and that one obtains for small $\epsilon = 4 - d$

$$(\Pi^{\mu\nu}(p^2))^{ab} = (p^2 \delta^{\mu\nu} - p^\mu p^\nu) \delta^{ab} \left(\frac{g^2}{(4\pi)^2} \left(-\frac{5}{3} \right) C_2(G) \frac{2}{\epsilon} + \mathcal{O}(\epsilon^0) \right).$$