

PROBLEMS FOR QUANTUM FIELD THEORY 2
3. Tutorial

PROBLEM 1: *Grassmann algebra*

Consider a set of Grassmann generators c_1, \dots, c_n . Show that the set \mathcal{G} of linear combinations

$$a = a^{(0)} + a_i^{(1)} c_i + a_{ij}^{(2)} c_i c_j + \dots + a^{(n)} c_1 c_2 \dots c_n$$

with complex numbers $a^{(0)}, a_i^{(1)}, \dots, a^{(n)}$ constitutes an algebra. To that end you have to show first that the set is a vector space over the complex numbers with the corresponding rules for addition and scalar multiplication. Thereafter you have to define an appropriate rule to multiply two elements of \mathcal{G} .

Which elements of the Grassmann algebra \mathcal{G} have an inverse?