Universität Heidelberg   SS 2010

Problems for Quantum Field Theory 2
4. Tutorial

Problem 1: *Legendre transform*
Consider a real function $f(x)$ of an real variable x. The Legendre transform is defined by

$$g(y) = \sup_x \left(xy - f(x) \right).$$

How can one construct the function $g(y)$ graphically? Do this for $f(x) = \lambda\, x^4 - \mu\, x^2$. What is the Legendre transform of $g(y)$?

Problem 2: *Effective action for Gaussian theory*
Consider a probabilistic theory for a N-dimensional vector $\mathbf{x}$. The partition function is given by

$$Z(\mathbf{J}) = e^{W(\mathbf{j})} = \int d\tilde{\mathbf{x}}\, e^{-S(\tilde{\mathbf{x}}) + J_n \tilde{x}_n}.$$

The effective action is defined as the Legendre transform of the Schwinger function $W(\mathbf{J})$,

$$\Gamma(\mathbf{x}) = \sup_{\mathbf{J}} \left(J_n x_n - W(\mathbf{J}) \right).$$

Calculate $\Gamma(\mathbf{x})$ for the Gaussian theory $S(\mathbf{x}) = \frac{1}{2} P_{mn} x_m x_n$.