Problems for Quantum Field Theory 2

6. Tutorial

See appendix A of chapter 4 of the script.

Problem 1: *Fadeev-Popov trick*
Consider a two-dimensional integral

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \; f(x_1^2 + x_2^2).$$

Note that the integral has a symmetry

$$x \to x_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} x.$$

Use the Faddev-Popov trick to remove this "gauge degree of freedom". To that end multiply the integral by a factor one in the form

$$1 = \int_{-\pi/2}^{\pi/2} d\alpha \; \delta(g(x_\alpha)) \left| \frac{\partial g(x_\alpha)}{\partial \alpha} \right|,$$

with an appropriate gauge-fixing function $g(x_\alpha)$. (A useful choice is $g(x_\alpha) = (x_\alpha)_2$.) You can now change the integration variables in the integral over x and integrate out one direction by using the δ-function.