

gauge theory

0-dim

$$\int dA e^{-S[A]}$$

$$\int d^2x e^{-S(x)}$$

$$u = e^{i\omega} \downarrow A^a = u A_a u^\dagger + \frac{1}{g} u \partial_\nu u^\dagger$$

$$R = \begin{pmatrix} \cos\theta & i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix} \downarrow x_1^\theta = x_1 \cos\theta + x_2 i\sin\theta = \vec{x}$$

$$1 = \int dg \delta[\mathcal{F}[A^a]] \cdot \Delta_{\mathcal{F}}[A]$$

$$1 = \int_0^{2\pi} d\theta \delta(x_1^\theta) \cdot \Delta_{\mathcal{F}}(\vec{x})$$

with

with

$$\Delta_{\mathcal{F}}[A] = \left(\int dg \delta[\mathcal{F}[A]] \right)^{-1}$$

$$\Delta_{\mathcal{F}}(\vec{x}) = \left(\int_0^{2\pi} d\theta \delta(x_1^\theta) \right)^{-1}$$

$$= \# \int_{\omega_0} d\omega \frac{\delta \mathcal{F}}{\delta \omega} \Big|_{A=A(\omega)}$$

$$= \left(\int_0^{2\pi} d\theta \frac{1}{|-x_1 \sin\theta + x_2 \cos\theta|} \right)$$

$$\left(\delta\left(\theta - \arctan \frac{x_2}{x_1}\right) + \delta\left(\theta - \arctan \frac{x_2}{x_1} + \pi\right) \right)$$

$$= \pi/2(\vec{x})$$

$$\int dA e^{-S[A]}$$

$$\Rightarrow \int d^2x e^{-S(x)}$$

$$= \int dA dg \delta[\mathcal{F}[A^a]] \Delta_{\mathcal{F}}[A] e^{-S[A]}$$

$$= \int d^2x \int_0^{2\pi} d\theta \delta(x_1^\theta) \frac{\pi}{2} e^{-S(x)}$$

$$= \int dA \delta[\tilde{\mathcal{F}}[A]] \Delta_{\mathcal{F}}[A] e^{-S[A]}$$

$$= \int d^2x \delta(x_1) \frac{\pi}{2} e^{-S(x)}$$

$$\cdot [\int dg]$$

$$\cdot [2\pi]$$

$$= \int dA_{gf} \Delta_{\mathcal{F}}[A_{gf}] e^{-S[A_{gf}]}$$

$$= \int_0^{\infty} dx_2 \cdot x_2 e^{-S(\sqrt{x_2^2})}$$

$$\cdot [\int dg]$$

$$\cdot [2\pi]$$

$$\text{with } x_2 = r \left(\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right)$$

Gribov copies:

$$x_2^\Theta = x_2 \cos \Theta - x_1 \sin \Theta \quad \left| \begin{array}{l} \Theta_+ = \arctan x_2/x_1 \\ \Theta_- = \arctan x_2/x_1 + \pi \end{array} \right. \quad x_2^{\Theta_+} = -x_2^{\Theta_-}$$

and $r = \pm x_2^\Theta$ ($x_2^{\Theta_+}$ can be posit./negative)

Remove absolute value:

$$|x_2^\Theta| \rightarrow x_2^\Theta$$

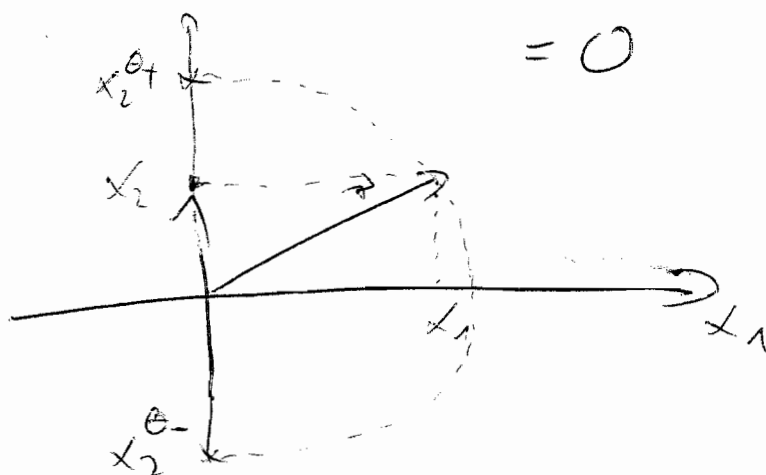
$$a.) \int d\Theta \frac{1}{x_2^\Theta} (\delta(\Theta - \arctan x_2/x_1) + \delta(\Theta - \arctan x_2/x_1 + \pi))$$

$$= 0$$

$$b.) \int d^2x \ x_2^\Theta / 2 \ \delta(x_1^\Theta) e^{-S(r)}$$

$$= \int d^2x \ x_2/2 \ \delta(x_1) e^{-S(r)} = \int dx_2 \ x_2/2 e^{-S(\sqrt{x_2^2})}$$

$$= 0$$



ghosts: assume exactly 1 copy

A-3

$$\boxed{\frac{\delta \mathcal{F}}{\delta \omega} = c}$$

$$\det \frac{\delta \mathcal{F}}{\delta \omega} = \int d\bar{c} dc e^{-\int d^d x \bar{c} \frac{\delta \mathcal{F}}{\delta \omega} c}$$

↑
II-37a

$$= \int d\bar{c} dc e^{-S_{gh}[A, c, \bar{c}]}$$

with

$$S_{gh}[A, c, \bar{c}] = \int d^d x \bar{c} \frac{\delta \mathcal{F}}{\delta \omega} c$$

cov. gauge

$$= - \int d^d x \bar{c}^a \partial_\nu \mathcal{D}_\nu^{ab} c^b$$

Generating functional: $\mathcal{F}^a \rightarrow \mathcal{F}^a - \mathcal{E}^a$
 $\Delta \mathcal{F} \rightarrow \Delta \mathcal{F}$

$$Z[J, \eta, \bar{\eta}] = \frac{1}{N} \int dA \int d\mathcal{E} e^{-\frac{1}{2\xi} \int d^d x \mathcal{E}^a(x)^2}$$

$$\int d\bar{c} dc e^{-S_{gh}[A, c, \bar{c}]}$$

$$\cdot \int d\mathcal{U} \delta[\mathcal{F} - \mathcal{E}] e^{-S_{YM}[A]}$$

$$\cdot e^{\int d^d x \{ \bar{\eta}^a A_\nu^a + \bar{c}^a \eta^a \}}$$

Remark II-37b: Neuberger Problem

Fermions - reminder:

Grassmann variables: (1) $c^2 = \bar{c}^2 = 0$

$$(2) c\bar{c} = -\bar{c}c$$

$$\int dc c^n = \delta_{n,1}$$

$$\Rightarrow \int dc f(c) = \left. \frac{\partial f}{\partial c} \right|_{c=0}$$

$$\Rightarrow \int \prod_n d\bar{c}_n dc_n e^{-\bar{c}_i M_{ij} c_j}$$

$$= \int \prod_{n=1}^N d\bar{c}_n dc_n e^{-\bar{c}_i M_{ij} c_j}$$

$$= \int \prod_{n=1}^N d\bar{c}_n dc_n \left(-\bar{c}_i M_{ij} c_j \right)^N \frac{1}{N!}$$

$$= \sum_{\sigma} (-1)^{\sigma} \prod_i M_{i\sigma(i)} = \det M$$

Neuberger problem:

(BRST)

$$\int dA_{gf} \det \frac{\delta \mathcal{F}}{\delta \psi} e^{-S[A_{gf}]} \stackrel{!}{=} 0$$

Resolutions:

(1) take absolute value:

difficult as gauge fixing is
done as extremisation

(2) gauge fixing as top. field theory
(Morse theory ...)

(3) do not gauge-fix