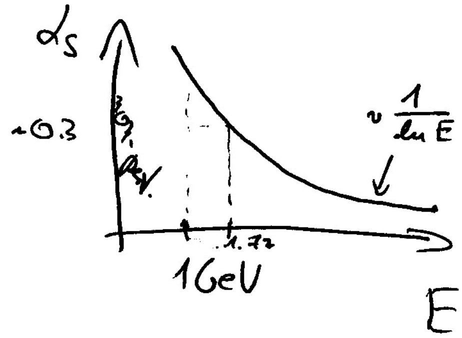
5 QCD

<u>Q</u>uantum <u>C</u>hromo <u>D</u>ynamics 'is' the theory of strong interactions. It has several peculiar properties

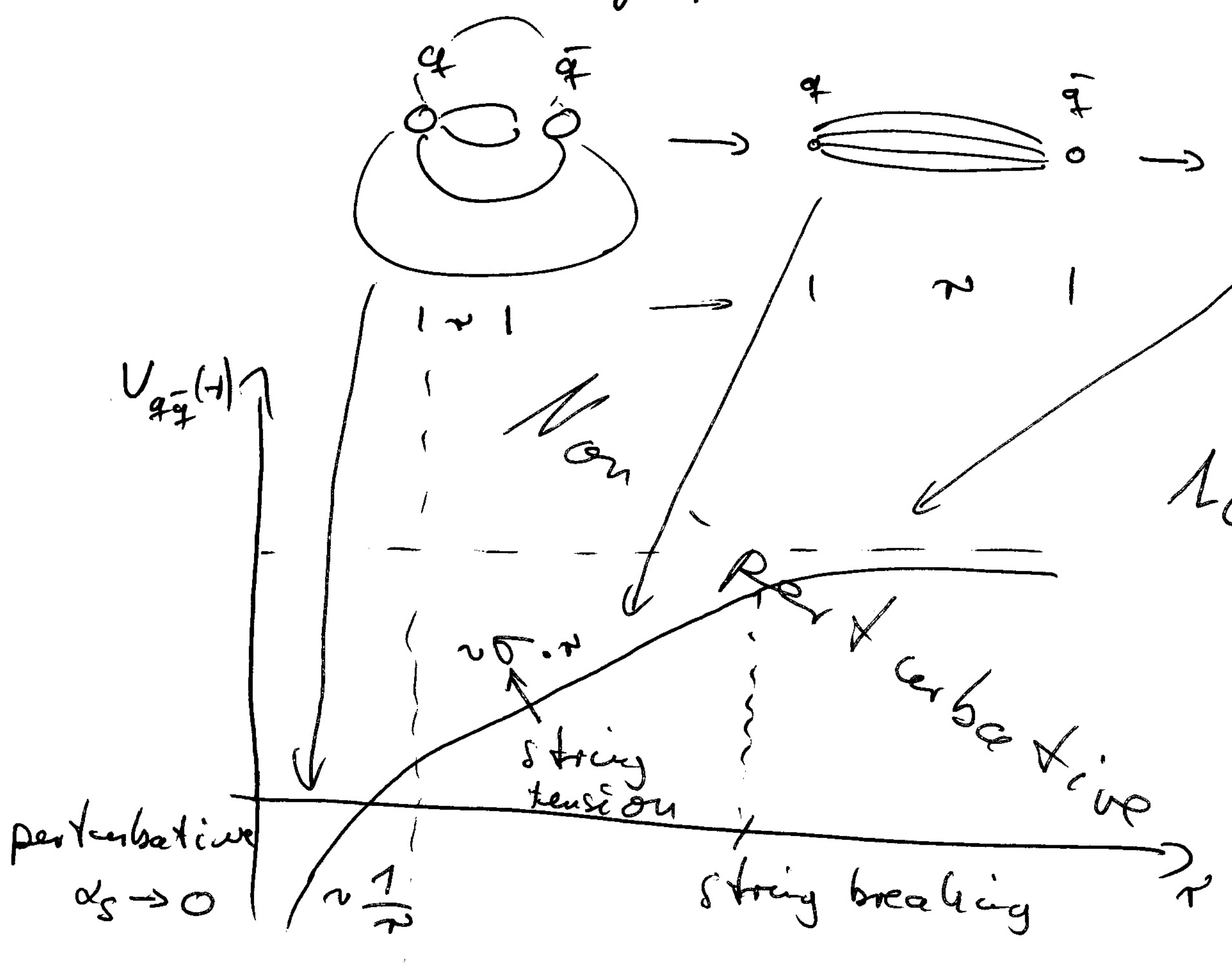
(a) asymptotic freedom



$\alpha_s = g^2/4\pi$: $\alpha_s(E \to \infty) \to 0$

(b) confinement

'no asymptotic coloured states'



$\Lambda_{QCD} \sim 3\text{-}400$ MeV

(c) spontaneous chiral symmetry breaking

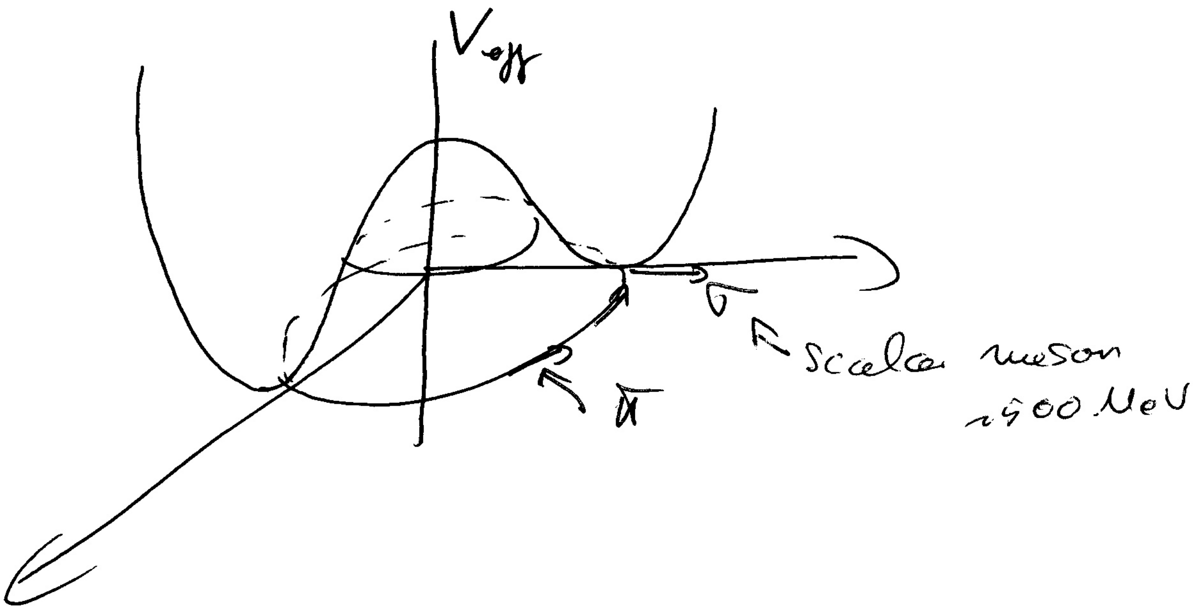
- quark masses:

light		heavy	correction		
u	d	s	c	b	t
1.5–3	3–7	95	1250	4200	$1.7\cdot 10^3$

Masses [MeV]

scale of chiral sym. breaking $\Delta m \sim 400\,\mathrm{MeV}$ flavour blind

- π is (pseudo-) Goldstone boson of chiral symm. breaking



$\pi^+ = u\bar{d}$

$\pi^- = -d\bar{u}$

$\pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$

$\vdots$

pseudo-scalar mesons

5.1 Renormalisation

QCD is a $SU(3)$ gauge theory (charge: colour) coupled to fermions in the fundamental representation (quarks): (Euclidean)

$$S[A,\psi,\bar\psi] = \int_x \left\{ \frac{1}{2} \operatorname{tr} F_{\mu\nu}^2 - \sum_f \bar\psi_f (\not{D} + m_f)\psi_f \right. \tag{5.1}$$

for details see p. (106a)

flavour

with flavours $f \in \{u,d,s,c,b,t\}$, $N_f = 6$.

In low energy QCD we use $N_f = \underset{\text{light}}{2} + \underset{\text{heavy}}{1}$.

Gauge field: $A_\mu = A_\mu^a t^a$ (5.2)

with $t^a = \frac{1}{2}\lambda^a$ ← Gell-Mann matrices

Gell-Mann matrices: (fund. representation)

$$\lambda^a = \begin{pmatrix} \sigma^a & 0 \\ 0 & 0 \end{pmatrix}, \quad a = 1,2,3 \tag{5.3}$$

$$\lambda^4 = \begin{pmatrix} & 0 & 1 \\ 1 & & \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} & 0 & -i \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^1 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^2 \end{pmatrix}$$

$$\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

For details see p. 65 – 68:

$$D_\mu = \partial_\mu - i g A_\mu$$

$$F_{\mu\nu} = \frac{i}{g} [D_\mu , D_\nu] \tag{5.1a}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

and

$$\begin{aligned} 1/4 \, F^a_{\mu\nu} F^a_{\mu\nu} = {} & \partial_\mu A^a_\nu \partial_\mu A^a_\nu - \partial_\mu A^a_\nu \partial_\nu A^a_\mu \\ & + g f^{abc} A^b_\mu A^c_\nu \partial_\mu A^a_\nu \\ & + g^2/4 \, f^{abc} f^{ade} A^b_\mu A^c_\nu A^d_\mu A^e_\nu \end{aligned} \tag{5.1b}$$

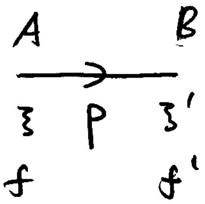
Cartan sub-algebra (max. Abelian sub-alg.)

$$[\lambda^3, \lambda^8] = 0 \qquad (5.4)$$

Feynman rules:

Pure glue: see p. 82 (5.5)

quarks: (p. 55-56)



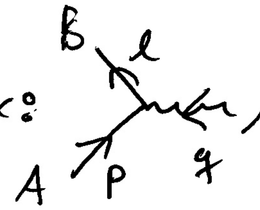
$$: \left[\frac{1}{i\not p + m}\right]_{\xi\xi'} \delta^{AB}\, \delta_{ff'}$$

gauge group (fund.)

Dirac

flavour

quark-gluon vertex:

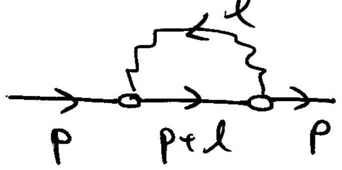


$$\nu, a : \; -i g\, \gamma_\nu (t^a)^{AB} \qquad (5.6)$$

Power counting:

Consider e.g. the self energy in 4d:

$\Sigma(p) =$



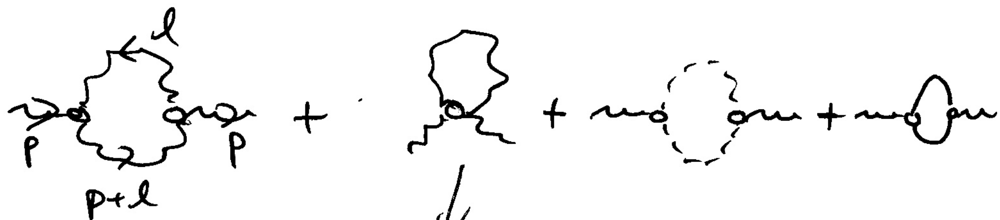
$$\sim \int \frac{d^4 l}{(2\pi)^4}\, t^a \gamma_\mu \frac{1}{i(\not p + \not l) + m}\, t^a \gamma_\nu \frac{1}{l^2}\left(\delta_{\mu\nu} - (1-\xi)\frac{l_\mu l_\nu}{l^2}\right) \qquad (5.7)$$

Remarks: Σ is linearly divergent

momentum counting:

$$\left.\begin{aligned} [d^4 l] &= 4 \\ \left[\frac{1}{i(\not p + \not l) + m}\right] &= -1 \\ \left[\frac{1}{l^2}\right] &= -2 \end{aligned}\right\} 4-1-2 = 1$$

Analogously: vacuum polarisation Π

$\Pi =$ 

$$[\Pi] = 2 \longleftarrow \sim \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2}$$

$\Rightarrow$ Highest divergence in Π potentially prod. a mass-term for the gluon.

$\Rightarrow$ $\boxed{\text{forbidden by gauge invariance}}$

$\Rightarrow$ STI or Master Equation

eq. (4.68), p. 88 eq. (4.99), p. 100

We have already seen that the master-equation implies transversality. More generally we write

$$\Gamma = \underset{\substack{\uparrow \\ \text{unrenormalised} \\ \text{classical action}}}{S} + \sum_{l=1}^{\infty} \underset{\substack{\nwarrow \\ \text{regularised} \\ l\text{-loop terms}}}{\Gamma_l} \tag{5.8}$$

We write the master equation eq. (4.99), p. 100 schematically as (also $(\Gamma,\Gamma)=0$)

$$\int_x \frac{\delta\Gamma}{\delta\phi}\cdot\frac{\delta\Gamma}{\delta L_\phi} = \boxed{\Gamma * \Gamma = 0} \leftarrow \text{gauge inv. regularisation} \tag{5.9}$$

$\phi = (A, c, \bar{c})$

Assume now that we have constructed a renormalised action from S (by redef. of fields & couplings, masses that renders the action finite at $l-1$ loop.

Then we have (only l-loop contr.)

$$S * \Gamma_l + \Gamma_l * S = -\sum_{n=1}^{l-1} \Gamma_n * \Gamma_{l-n} \tag{5.10}$$

The rhs of eq. (5.10) is finite by induction.

Hence we have

$$\boxed{S * \Gamma_l^{div} + \Gamma_l^{div} * S = 0} \qquad (5.11)$$

Now we define

$$S_l = S_{l-1} - \Gamma_l^{div} + O(l+1\text{-loop}) \quad (5.12)$$

wich renders the effective action finite.

Remarks:

(1) Renormalisability: Γ_l^{div} local
'proof' later

(2) (1) and (5.11) imply that
$\Gamma_l^{div} \sim S$: Γ_l^{div} can be absorbed in multiplicative factors in the classical action and BRST-variations.

Renormalisation scheme:

Analogously to chapter 7, p. 177–, QFT I we define the bare action: $S = S[A,\psi,\bar\psi] + S_{gf}[A,c,\bar c] + S_{gf}[A]$

$$S[A_0, c_0, \bar c_0, \psi_0, \bar\psi_0; g_0, m_0, \xi_0] \qquad (5.13)$$

$$= S[Z_A^{1/2} A, Z_c^{1/2} c, Z_c^{1/2} \bar c, Z_\psi^{1/2}\psi; Z_g g, Z_m m, Z_\xi \xi]$$

$\phi_0 = Z_\phi^{1/2}\phi$, $\quad g_0 = Z_g g$, $m_0 = Z_m m$, $\xi_0 = Z_\xi \xi$

The BRST-identity eq. (408) for the gluon 2-point fct. reduced to the local divergent part implies

$$\boxed{Z_\xi = Z_A} \qquad (5.14)$$

Common notation:

$$\begin{aligned}
&Z_3 (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 &&: \quad Z_3 = Z_A \\
&\tilde Z_3 \; \bar c\, \partial^2 c &&: \quad \tilde Z_3 = Z_c \qquad (5.15)\\
&Z_1 \; \partial A\, A^2 &&: \quad Z_1 = Z_g \cdot Z_A^{3/2} \\
&Z_4 \; A^4 &&: \quad Z_4 = Z_g^2 Z_A^2 \\
&\tilde Z_1 \; \bar c\, \partial A c &&: \quad \tilde Z_1 = Z_g Z_c Z_A^{1/2}
\end{aligned}$$

with the pure YM relation

$$\frac{Z_4}{Z_1} = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = Z_g Z_A^{1/2} \qquad (5.16)$$

and, with

$$Z_2 \ \bar{\psi} \not{\partial} \psi : \quad Z_2 = Z_\psi$$
$$Z_{1,F} \ \bar{\psi} \not{A} \psi : \quad Z_{1,F} = Z_g Z_A^{1/2} Z_\psi \qquad (5.17)$$

it also follows that

$$Z_g Z_A^{1/2} = \frac{Z_{1,F}}{Z_2} \qquad (5.18)$$

All Z's have an expansion in powers of g:

$$1 + \delta Z = Z = 1 + g^2(\text{div} + \text{finite}) + g^4(\text{div} + \text{finite}) + \dots \qquad (5.19)$$

In the following we will use dimensional regularisation: $d = 4 - 2\varepsilon$ in d dimensions.

gauge invariant and $\text{div} = \#/\varepsilon$ see QFT I, p. 187

$Z = 1 + \delta Z$

'Minimal subtraction': $\boxed{Z = 1 + \#/\varepsilon}$ (5.20)

Remarks: (1) momentum cut-off <u>not</u> gauge inv.

(2) Other Reg. than dim-reg: $MS \neq \{Z = 1 + \text{div}\}$