7.3 Callan-Symanzik equation

Let us first come back to our generating functional Z with

$$Z = \int [D\phi]_\Lambda \, e^{-S_{eff,\Lambda}[\phi]} \qquad (7.46)$$

Note that Z is Λ-independent

$$\boxed{\Lambda \frac{d}{d\Lambda} Z = 0} \qquad (7.47)$$

as it is simply $= \int D\phi \, e^{-S[\phi]}$. Using the β-functions as defined in eq. (7.30), eq. (7.45) can be rewritten as

no hat, dim. fcel

$$\boxed{\left(\Lambda \frac{\partial}{\partial \Lambda} + \beta_i \frac{\partial}{\partial \lambda_i} \right) Z = 0} \qquad (7.48)$$

which entails the RG-invariance / regularisation independence of the theory, with

$$\beta_i = \Lambda \frac{d\lambda_i}{d\Lambda} \qquad (7.49)$$

Even though we have performed our derivation for Z, it also hold true for

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = G^{(n)}(x_1, \dots, x_n) \quad (7.50)$$

renormalised Green fct.

However, the reparameterisation step $\phi \to \frac{1}{Z^{1/2}} \phi$ causes a global Λ-dependence of eq. (7.50) and hence

$$\left(\Lambda \frac{d}{d\Lambda} + n/2 \underbrace{\frac{1}{Z} \frac{dZ}{d\Lambda}}_{\gamma} \right) G^{(n)} = 0 \quad (7.51)$$

The total Λ-derivative can be rewritten in terms of the β-fcts.:

$$\left(\Lambda \frac{\partial}{\partial \Lambda} + \beta_i \frac{\partial}{\partial \lambda_i} + n/2\, \gamma \right) G^{(n)} = 0 \quad (7.52)$$

We remark that we also could have started with the unrenormalised correlation function $G_0^{(n)} = \langle \phi_0 \cdots \phi_0 \rangle$ (7.53)

and, by varying the RG-scale μ, at which the RG-conditions are fixed (see QFT I), together with using that $\phi_{ren} = \phi_0 \frac{1}{Z^{1/2}}$, we would have arrived at eq. (7.52) with $\Lambda \to \mu$.
Thus we can map the task of computing β_i to that of computing $\Delta Z, \Delta m^2, \dots$, eg., in dim. reg..
However, beyond perturbation theory, Wilson's approach is far more powerful also for practical purposes.