
Quantum Field Theory 2 – Problem set 3

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Suggested reading before solving these problems: Chapter 1, 2.1 in the script and/or chapter 9.5, 11.3 –11.5 in *Peskin & Schroeder*.

Problem 1: Effective potential

In the lecture you derived an expression for the effective action in one-loop approximation

$$\Gamma_{1\text{-loop}}[\phi] = S[\phi] + \frac{1}{2} \text{Tr} \ln S^{(2)}[\phi].$$

Consider ϕ^4 theory in d Euclidean dimensions

$$S[\phi] = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu \partial_\mu) \phi + V_0(\phi) \right\}, \quad (1)$$

with $V_0(\phi) = \frac{1}{2} m_0^2 \phi^2 + \frac{1}{4!} \lambda_0 \phi^4$. Write the effective action in terms of an derivative expansion

$$\Gamma_{1\text{-loop}}[\phi] = \int d^d x \left\{ \frac{1}{2} \phi (-Z_1 \partial_\mu \partial_\mu + Z_2 \partial_\mu \partial_\mu \partial_\nu \partial_\nu + \dots) \phi + V_{\text{eff}}(\phi) \right\} + \text{const.} \quad (2)$$

The first line vanishes for constant fields

$$\Gamma_{1\text{-loop}}[\phi] \Big|_{\phi=\text{const}} = \int d^d x V_{\text{eff}}(\phi) + \text{const.}$$

The functional trace operation Tr is best evaluated in momentum space

$$\phi(x) = \int \frac{d^d p}{(2\pi)^d} \phi(p) e^{ipx}. \quad (3)$$

a) Show that the effective potential is given by

$$V_{\text{eff}}(\phi) = V_0(\phi) + \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \ln \left(p^2 + m_0^2 + \frac{\lambda_0}{2} \phi^2 \right) + \text{const.} \quad (4)$$

- b) It is often useful to expand the effective potential in a Taylor series. A discrete symmetry $\phi \rightarrow -\phi$ implies that only even terms appear

$$V_{\text{eff}}(\phi) = V_{\text{eff}}(0) + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 + \dots \quad (5)$$

Derive formal expressions in terms of integrals for m^2 and λ .

- c) The effective action $\Gamma[\phi]$ can be used to generate the amputated n -point functions. Calculate the Feynman diagrams up to 1-loop order for the two- and four-point functions at vanishing external momenta. Show that one obtains indeed the same expressions as derived in part b).

Problem 2: Grassmann variables: Change of variables

Consider a set of Grassmann generators c_1, \dots, c_n and a non-singular matrix A . For $c'_i = A_{ij}c_j$ prove

$$c'_1 \dots c'_n = (\det A) c_1 \dots c_n.$$

Show that this implies

$$\int dc_1 \dots dc_n = \int dc'_1 \dots dc'_n J, \quad \text{with} \quad J = \det \frac{\partial c'_i}{\partial c_j}.$$