Quantum Field Theory 2 – Problem set 7

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Suggested reading before solving these problems: Chapter 4.3 in the script and/or chapter 16.1-16.4 in *Peskin & Schroeder*.

Problem 1: BRST Symmetry

Consider the action of a Non-Abelian gauge theory after gauge fixing with the Fadeev-Popov method

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 - \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\}, \tag{1}$$

with $D_\mu = \partial_\mu - igA_\mu$.

a) Show that eq. (1) is equivalent to

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 - \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \frac{\xi}{2} b^a b^a + b^a \partial_\mu A^a_\mu - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\}, \tag{2}$$

if the equation of motion for b is used. Note that b^a is an auxiliary field (Nakanishi-Laudrup field). It can be eliminated by solving its field equation or equivalently by performing the Gaussian integral over it.

b) Show that the action in eq. (2) is invariant under the contiuous symmetry

$$\begin{aligned} \delta_\epsilon A^a_\mu &= \epsilon\, D^{ac}_\mu c^c, \\ \delta_\epsilon \psi &= ig\epsilon\, c^a t^a \psi, \\ \delta_\epsilon c^a &= -\frac{1}{2} g\epsilon\, f^{abc} c^b c^c, \\ \delta_\epsilon \bar{c}^a &= \epsilon\, b^a, \\ \delta_\epsilon b^a &= 0, \end{aligned}$$

with infinitesimal Grassmann-valued parameter ϵ. Tip: rewrite the BRST-transformations as matrix equations, i.e. $\delta_\epsilon A_\mu = \epsilon D_\mu c$, $\delta_\epsilon \psi = ig\epsilon\psi$, $\delta_\epsilon c = ig\epsilon c^2$, $\delta_\epsilon \bar{c} = \epsilon b$.

c) The BRST charge operator Q is defined by $\delta_\epsilon \phi = \epsilon Q \phi$, where ϕ is one of the fields A^a_μ, ψ, c^a, $\bar{c}^a$ and b^a. Show that

$$Q^2 \phi = 0.$$

d) Use the BRST charge operator Q to devide the Hilbert space into three distinct subspaces and identify the physical Hilbert space.