Quantum Field Theory 2 – Problem set 9

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Suggested reading before solving these problems: Chapter 5.1-5.2 in the script and/or chapter 16.5 in *Peskin & Schroeder*.

Problem 1: Gauge-Boson self energy

Consider the self energy of the gauge bosons in pure Yang-Mills theory (i.e. only glouns and ghosts). Due to BRST invariance it is of the form

$$\left(\Pi^{\mu\nu}(p^2)\right)^{ab} = (p^2\delta^{\mu\nu} - p^\mu p^\nu)\Pi(p^2)^{ab}.$$

Show that there are three diagrams contributing at one-loop level. Show that the corresponding expressions are in Feynman gauge ($\xi = 1$)

$$\left(\Pi_1^{\mu\nu}(p^2)\right)^{ab} = \frac{g^2}{2}\int \frac{d^4l}{(\pi)^4}\frac{1}{l^2(l+p)^2} f^{acd} f^{bdc} N^{\mu\nu}$$

with

$$\begin{aligned} N^{\mu\nu} =& [\delta^{\mu\rho}(p-l)^\sigma + \delta^{\rho\sigma}(2l+p)^\mu + \delta^{\sigma\mu}(-l-2p)^\rho] \\ & [\delta^{\nu\rho}(l-p)^\sigma + \delta^{\rho\sigma}(-2l-p)^\nu + \delta^{\nu\sigma}(l+2p)^\rho], \end{aligned}$$

and

$$\left(\Pi_2^{\mu\nu}(p^2)\right)^{ab} = \frac{g^2}{2}\int \frac{d^4l}{(2\pi)^4}\frac{\delta^{\rho\sigma}}{l^2}\delta^{cd}(M^{\rho\sigma})^{ab\,cd},$$

with

$$\begin{aligned} (M^{\rho\sigma})^{ab\,cd} =& f^{abe} f^{cde}(\delta^{\mu\rho}\delta^{\nu\sigma} - \delta^{\mu\sigma}\delta^{\nu\rho}) + f^{ace} f^{bde}(\delta^{\mu\nu}\delta^{\rho\sigma} - \delta^{\mu\sigma}\delta^{\nu\rho}) \\ & + f^{ade} f^{bce}(\delta^{\mu\nu}\delta^{\rho\sigma} - \delta^{\mu\rho}\delta^{\nu\sigma}), \end{aligned}$$

as well as

$$\left(\Pi_3^{\mu\nu}(l^2)\right)^{ab} = (-1)g^2\int \frac{d^4l}{(2\pi)^4}\frac{1}{l^2(l+p)^2} f^{dac}(l+p)^\mu f^{cbd} l^\nu.$$

The expressions for $\Pi_1^{\mu\nu}$, $\Pi_2^{\mu\nu}$ and $\Pi_3^{\mu\nu}$ can be simplified further by using dimensional regularization, introducing Feynman parameters and by using the identity

$$f^{acd} f^{bcd} = C_2(G)\,\delta^{ab}.$$

Show that the sum $\Pi^{\mu\nu} = \Pi_1^{\mu\nu} + \Pi_2^{\mu\nu} + \Pi_3^{\mu\nu}$ is finite for $d \to 2$ and that one obtains for small $\epsilon = 4 - d$

$$\left(\Pi^{\mu\nu}(p^2)\right)^{ab} = (p^2\delta^{\mu\nu} - p^\mu p^\nu)\delta^{ab}\left(\frac{g^2}{(4\pi)^2}\left(-\frac{5}{3}\right)C_2(G)\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right).$$