Quantum Field Theory 2 – Problem set 2

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Problem 1: Partition function for imaginary temperature

Consider the canonical partition function

$$Z = \operatorname{Tr} e^{-\beta H} = \sum_n \langle n | e^{-\beta H} | n \rangle$$

where $|n\rangle$ is an arbitrary basis and $\beta = 1/T$. Show that this can be written as

$$Z = \int dq_0 \langle q_0 | e^{-\beta H} | q_0 \rangle$$

where $|q_0\rangle$ labels eigenstates of the position operator. Let us assume for the moment that the temperature is imaginary so that we can write

$$\beta = it$$

with $t > 0$. Use the path integral formula for the transition amplitude

$$\langle q_f | e^{-iHt} | q_i \rangle = \int_{q(0)=q_i, q(t)=q_f} Dq \; e^{iS[q]},$$

$$S[q] = \int_0^t dt' \; L(q, \dot{q}, t')$$

to show that Z can be written as a functional integral over periodic functions $q(t')$ with $q(t) = q(0)$.