Quantum Field Theory 2 – Tutorial 3

Lectures: Jan Pawlowski Tutorials: Eduardo Grossi Institut für Theoretische Physik, Uni Heidelberg

j.pawlowski@thphys.uni-heidelberg.de e.grossi@thphys.uni-heidelberg.de due date: 2 May 2017

Problem 1: Grassmann algebra

Consider a set of Grassmann generators c_1, \ldots, c_n . Show that the set \mathcal{G} of linear combinations

 $a = a^{(0)} + a_i^{(1)}c_i + a_{ij}^{(2)}c_ic_j + \dots + a^{(n)}c_1c_2\dots c_n$

with complex numbers $a^{(0)}, a_i^{(1)}, \ldots, a^{(n)}$ constitutes an algebra. To that end you have to show first that the set is a vector space over the complex numbers with the corresponding rules for addition and scalar multiplication. Thereafter you have to define an appropriate rule to multiply two elements of \mathcal{G} .

Which elements of the Grassmann algebra \mathcal{G} have an inverse?