Quantum Field Theory 2 – Tutorial 8

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Problem 1: BRST transformation of the anti-ghost

Consider the action of a Non-Abelian gauge theory after gauge fixing with the Faddeev-Popov method

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\} ,$$

with $D_\mu = \partial_\mu - igA_\mu$. Show that the action is invariant under the contiuous symmetry

$$\begin{aligned} \delta_\epsilon A^a_\mu &= \epsilon\, D^{ac}_\mu c^c, \\ \delta_\epsilon c^a &= -\frac{1}{2} g\epsilon\, f^{abc} c^b c^c, \\ \delta_\epsilon \bar{c}^a &= \epsilon \frac{1}{\xi} \partial_\mu A^a_\mu, \end{aligned}$$

with infinitesimal Grassmann-valued parameter ϵ. The BRST charge operator Q is defined by

$$\delta_\epsilon \phi = \epsilon Q \phi\,, \qquad \text{with} \qquad \phi = (A^a_\mu, c^a, \bar{c}^a)\,.$$

Show that Q is not nilpotent in this case,

$$Q^2 \phi \neq 0\,,$$

and in particular

$$Q^2 \bar{c} \neq 0\,.$$