Quantum Field Theory 2 – Tutorial 9

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Problem 1: Casimir operator

Consider a Lie algebra with commutation relation

$$[T^a, T^b] = if^{abc}T^c.$$

Show that the operator $T^2 = T^aT^a$ commutes with all generators, i.e.

$$[T^b, T^aT^a] = 0\,.$$

This implies that T^2 takes a constant value for each irreducible representation t_r^a and the matrix representation is proportional to the unit matrix

$$t_r^a t_r^a = C_2(r) \cdot \mathbb{1}\,.$$

Use the normalization

$$\mathrm{tr}[t_r^a t_r^b] = C(r)\delta^{ab}\,,$$

to show the identity

$$d(r)C_2(r) = d(G)C(r)\,,$$

where $d(r)$ is the dimension of the (matrix) representation r and $d(G)$ is the dimension of the adjoint representation.