Quantum Field Theory 2 – Problem set 5

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Suggested reading before solving these problems: Chapter 3.1-3.2 in the script and/or chapter 7.5 in *Peskin & Schroeder*.

Problem 1: The exact two-point function

a) Show diagrammatically that one can write the renormalized two-point function of some theory as

$$G(p) = G_0(p) + G_0(p)\Pi(p)G_0(p) + G_0(p)\Pi(p)G_0(p)\Pi(p)G_0(p) + \dots, \qquad (1)$$

where $G_0(p)$ is the free two-point function and $\Pi(p)$ is the sum of all one-particle irreducible insertions into the propagator.

b) Show that the above series can be resumed

$$G(p) = \left(G_0^{-1}(p) - \Pi(p)\right)^{-1}. \qquad (2)$$

c) Consider now a real scalar field φ in Euclidean space. What is the relation of $\Pi(p)$ to the renormalized mass $m^2_{\varphi,r}$ and the wavefunction renormalization constant Z_φ defined by the expansion

$$G^{-1}(p) = Z_\varphi \left(p^2 + m^2_{\varphi,r}\right) + \mathcal{O}(p^4) \qquad (3)$$

for small values of p^2?

Problem 2: Two point function of Yukawa theory

On the previous exercise sheet you have derived a perturbative expression for the boson two-point function of the Yukawa theory in d Euclidean dimensions

$$G(p) = G_0(p) + G_0(p)\Pi(p)G_0(p) + \dots,$$

where

$$\Pi(p) = -h^2 \int \frac{d^dq}{(2\pi)^d} \frac{-d\, q\cdot(q+p) + d\, m^2}{(q^2+m^2)((q+p)^2+m^2)},$$

with m being the mass of the fermions. Simplify the expression of $\Pi(p)$ further by introducing the Feynman parameters (see also the Problem 2 in the Sheet 9 of QFT I) and derive the expressions for the renormalized mass $m^2_{\varphi,r}$ and the wavefunction renormalization constant Z_φ at the leading order in the large parameter $1/\epsilon$, with $\epsilon = 4 - d \ll 1$.