Quantum Field Theory 2 – Problem set 7

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Suggested reading before solving these problems: Chapter 13.3 in the script and/or chapter 16.1-16.4 in *Peskin & Schroeder*.

Problem 1: BRST symmetry

Consider the action of a non-Abelian gauge theory including the gauge-fixing term via the Faddeev-Popov method:

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 - \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\} , \tag{1}$$

with $D_\mu = \partial_\mu - igA_\mu$.

a) Show that the action in Eq. (1) is equivalent to the action

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 - \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \frac{\xi}{2} b^a b^a + b^a \partial_\mu A^a_\mu - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\} \tag{2}$$

upon employing the equation of motion for the field b^a. Note that b^a is an auxiliary field (the Nakanishi-Lautrup field) that can be eliminated by solving its field equation or, equivalently, by performing the Gaussian integral over it.

b) Show that the action in Eq. (2) is invariant under the continuous transformation (BRST transformation)

$$\begin{aligned} \delta_\epsilon A^a_\mu &= \epsilon\, D^{ac}_\mu c^c, \\ \delta_\epsilon \psi &= ig\epsilon\, c^a t^a \psi, \\ \delta_\epsilon c^a &= -\frac{1}{2} g\epsilon\, f^{abc} c^b c^c, \\ \delta_\epsilon \bar{c}^a &= \epsilon\, b^a, \\ \delta_\epsilon b^a &= 0, \end{aligned} \tag{3}$$

with the infinitesimal Grassmann-valued parameter ϵ.

Hint: rewrite the BRST-transformations as matrix equations, i.e., $\delta_\epsilon A_\mu = \epsilon D_\mu c$, $\delta_\epsilon \psi = ig\epsilon c\psi$, $\delta_\epsilon c = ig\epsilon c^2$, $\delta_\epsilon \bar{c} = \epsilon b$.

c) The BRST charge operator Q is defined as the generator of the BRST transformation,

$$\delta_\epsilon \phi = \epsilon Q \phi, \tag{4}$$

where ϕ is one of the fields A^a_μ, ψ, c^a, $\bar{c}^a$, and b^a. Show that

$$Q^2\phi = 0. \tag{5}$$

d) Use the BRST charge operator Q to divide the Hilbert space into three distinct subspaces and identify the physical Hilbert space.