Quantum Field Theory 2 – Problem set 9

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Suggested reading before solving these problems: Chapter 14.1-14.2 in the script and/or chapter 16.5 in *Peskin & Schroeder*.

Problem 1: Gauge boson self-energy

Consider the self-energy of gauge bosons in pure Yang-Mills theory (i.e., including only gluons and ghosts). Due to the BRST invariance, it has the form

$$\left(\Pi^{\mu\nu}(p^2)\right)^{ab} = (p^2\delta^{\mu\nu} - p^\mu p^\nu)(\Pi(p^2))^{ab}. \tag{1}$$

Show that there are three diagrams contributing to the self-energy at one-loop level and that the corresponding expressions in the Feynman gauge ($\xi = 1$) read

$$\left(\Pi_1^{\mu\nu}(p^2)\right)^{ab} = \frac{g^2}{2}\int \frac{d^4l}{(2\pi)^4}\frac{1}{l^2(l+p)^2} f^{acd}f^{bcd}N^{\mu\nu}, \tag{2a}$$

$$\left(\Pi_2^{\mu\nu}(p^2)\right)^{ab} = \frac{g^2}{2}\int \frac{d^4l}{(2\pi)^4}\frac{\delta^{\rho\sigma}}{l^2}\delta^{cd}(M^{\rho\sigma})^{ab\ cd}, \tag{2b}$$

$$\left(\Pi_3^{\mu\nu}(p^2)\right)^{ab} = (-1)g^2\int \frac{d^4l}{(2\pi)^4}\frac{1}{l^2(l+p)^2} f^{dac}(l+p)^\mu f^{cbd}l^\nu, \tag{2c}$$

with

$$\begin{aligned} N^{\mu\nu} =& [\delta^{\mu\rho}(p-l)^\sigma + \delta^{\rho\sigma}(2l+p)^\mu + \delta^{\sigma\mu}(-l-2p)^\rho] \\ & [\delta^{\nu\rho}(l-p)^\sigma + \delta^{\rho\sigma}(-2l-p)^\nu + \delta^{\nu\sigma}(l+2p)^\rho] \end{aligned} \tag{3}$$

and

$$\begin{aligned} (M^{\rho\sigma})^{ab\ cd} =& f^{abe}f^{cde}(\delta^{\mu\rho}\delta^{\nu\sigma} - \delta^{\mu\sigma}\delta^{\nu\rho}) + f^{ace}f^{bde}(\delta^{\mu\nu}\delta^{\rho\sigma} - \delta^{\mu\sigma}\delta^{\nu\rho}) \\ & + f^{ade}f^{bce}(\delta^{\mu\nu}\delta^{\rho\sigma} - \delta^{\mu\rho}\delta^{\nu\sigma}), \end{aligned} \tag{4}$$

The expressions for $\Pi_1^{\mu\nu}(p^2)$, $\Pi_2^{\mu\nu}(p^2)$, and $\Pi_3^{\mu\nu}(p^2)$ can be simplified further by employing dimensional regularization, introducing Feynman parameters, and using the identity

$$f^{acd}f^{bcd} = C_2(G)\,\delta^{ab}. \tag{5}$$

Show that the sum $\Pi^{\mu\nu}(p^2) = \Pi_1^{\mu\nu}(p^2) + \Pi_2^{\mu\nu}(p^2) + \Pi_3^{\mu\nu}(p^2)$ is finite for $d \to 2$ and that for small $\epsilon = 4 - d$, it is given by

$$\left(\Pi^{\mu\nu}(p^2)\right)^{ab} = \left(p^2\delta^{\mu\nu} - p^\mu p^\nu\right)\delta^{ab}\left[\frac{g^2}{(4\pi)^2}\left(-\frac{5}{3}\right)C_2(G)\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right]. \tag{6}$$