Quantum Field Theory 2 – Tutorial 4

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Problem 1: Legendre transform

Consider a real function $f(x)$ of a real variable x. The Legendre transform of $f(x)$ is defined by

$$g(y) = \sup_x \left(xy - f(x)\right). \tag{1}$$

How can one construct the function $g(y)$ graphically? Do this for $f(x) = \lambda\, x^4 - \mu\, x^2$. What is the Legendre transform of $g(y)$?

Problem 2: Effective action for Gaussian theory

Consider a probabilistic theory for a N-dimensional vector $\boldsymbol{x}$. The partition function is given by

$$Z(\boldsymbol{J}) = e^{W(\boldsymbol{j})} = \int d^N\tilde{x}\; e^{-S(\tilde{\boldsymbol{x}})+J_n\tilde{x}_n}. \tag{2}$$

The effective action is defined as the Legendre transform of the Schwinger function $W(\boldsymbol{J})$,

$$\Gamma(\boldsymbol{x}) = \sup_{\boldsymbol{J}} \left(J_n x_n - W(\boldsymbol{J})\right). \tag{3}$$

Calculate $\Gamma(\boldsymbol{x})$ for the Gaussian theory $S(\boldsymbol{x}) = \frac{1}{2}P_{mn}x_m x_n$.