
Quantum Field Theory 2 – Tutorial 6

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Problem 1: Faddeev-Popov trick

Consider a two-dimensional integral

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 f(x_1^2 + x_2^2). \quad (1)$$

Note that the integral has a symmetry

$$x \rightarrow x_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} x. \quad (2)$$

Use the Faddeev-Popov trick to remove this “gauge redundancy”. To that end multiply the integral by a factor one in the form

$$1 = \int_{-\pi/2}^{\pi/2} d\alpha \delta(g(x_\alpha)) \left| \frac{\partial g(x_\alpha)}{\partial \alpha} \right|, \quad (3)$$

with an appropriate gauge-fixing function $g(x_\alpha)$. (A useful choice is $g(x_\alpha) = (x_\alpha)_2$.) You can now change the integration variables in the integral over x and integrate out one direction by using the δ -function.