Quantum Field Theory 2 – Tutorial 8

Lectures: Jan Pawlowski J.Pawlowski@thphys.uni-heidelberg.de
Tutorials: Aleksandr Mikheev A.Mikheev@thphys.uni-heidelberg.de
Institut für Theoretische Physik, Uni Heidelberg tutorial date: week of 05.06.2023

Problem 1: BRST transformation of the anti-ghost

Consider the action of a non-Abelian gauge theory after gauge fixing using the Faddeev-Popov method

$$S = \int_x \left\{ \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 + \frac{1}{2\xi} \left(\partial_\mu A^a_\mu \right)^2 - \bar{c}^a \partial_\mu D^{ac}_\mu c^c \right\} , \tag{1}$$

with $D_\mu = \partial_\mu - igA_\mu$. Show that the action is invariant under the continuous symmetry

$$\begin{aligned} \delta_\epsilon A^a_\mu &= \epsilon\, D^{ac}_\mu c^c, \\ \delta_\epsilon c^a &= -\frac{1}{2} g\epsilon\, f^{abc} c^b c^c, \\ \delta_\epsilon \bar{c}^a &= \epsilon \frac{1}{\xi} \partial_\mu A^a_\mu, \end{aligned} \tag{2}$$

with the infinitesimal Grassmann-valued parameter ϵ. The BRST charge operator Q is defined by

$$\delta_\epsilon \phi = \epsilon Q \phi \, , \tag{3}$$

with $\phi = (A^a_\mu, c^a, \bar{c}^a)$. Show that Q is not nilpotent in this case,

$$Q^2 \phi \neq 0 \, , \tag{4}$$

and in particular

$$Q^2 \bar{c} \neq 0 \, . \tag{5}$$