
Quantum Field Theory 2 – Tutorial 11

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tutorial date: week of 26.06.2023

Problem 1: Casimir operator 3

The $SU(2)$ generators in the fundamental representation are directly related to the Pauli matrices,

$$t_2^a = \frac{\sigma^a}{2}. \quad (1)$$

They satisfy the normalization condition $\text{tr}[t_2^a t_2^b] = \frac{1}{2} \delta^{ab} = C(2) \delta^{ab}$. What is the value of $C_2(2)$ of the quadratic Casimir element $t_2^a t_2^a = C_2(2) \cdot \mathbb{1}$?

Hint: Use the identity $d(r)C_2(r) = d(G)C(r)$.

Likewise, for $SU(N)$, the generators in the fundamental representation (N -dimensional) satisfy the identities

$$\text{tr}[t_N^a t_N^b] = \frac{1}{2} \delta^{ab}. \quad (2)$$

What is the value of $C_2(N)$?