## **Quantum Field Theory 2 – Tutorial 12**

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## **Problem 1: Group integration**

Consider the group SU(3). Since this group is compact, it is possible to properly define an integration over the group elements. While the choice of measure is not unique, there exists a measure definition that plays a distinguished role, the so-called Haar measure. For compact Lie groups, it is invariant under both left and right multiplications, i.e.,

$$\forall V \in SU(3) \implies \int dUf(VU) = \int dUf(UV) = \int dUf(U), \tag{1}$$

a property crucial for preserving the path integral's gauge invariance. Assuming that the measure is normalized to 1,

$$\int dU \ 1 = 1,\tag{2}$$

prove the following integration rules:

$$\int dU \ U^{ab} = 0, \tag{3a}$$

$$\int dU \ U^{ab} U^{cd} = 0, \tag{3b}$$

$$\int dU \ U^{ab} (U^{\dagger})^{cd} = \frac{1}{3} \delta_{ad} \delta_{bc}.$$
(3c)